



Probability Teaching at School Level in China

WU Yingkang

East China Normal University

ykwu@math.ecnu.edu.cn



Outline

- ❖ Curriculum Requirement of probability
- ❖ Representation of probability content in textbooks
- ❖ Classroom teaching of probability
- ❖ Assessment of probability content

Curriculum Requirement

- ❖ Probability is not taught in schools in most part of China before 2001.
- ❖ The content of statistics and probability is highly valued in the new mathematics curriculum standard which was released in 2001.

The requirements of probability in the new mathematics curriculum (Grade 1~Grade 9)

Stage I (grades 1~3)	Stage II (grades 4~6)	Stage III (grades 7~9)
<ul style="list-style-type: none">- Have an initial experience that the happening of some events is certain, whereas some are uncertain.- Able to list all possible outcomes of simple experiments.- Know that the possibility of an event happened can be large or small.- Able to describe the possibility of happening of simple events.	<ul style="list-style-type: none">- Experience equal possibility of events happened, and fairness of the rule of the game. Able to obtain the possibility of simple events happened.- Design a project that meets specified requirements.- Make predictions of the possibility of a simple event happened, and give reasons.	<ul style="list-style-type: none">- Understand the meaning of probability in concrete situations; Deploy enumeration (including tabulation, drawing tree diagram) to calculate the probability of simple event happened.- Obtain the probability (frequency) of an event happened through experimentation. Know that rate (frequency) can be used as an estimated value to indicate the probability of an event happened when a large number of experiments are replicated.- Through real examples move a step further to enrich one's knowledge on probability, and is able to solve some practical problems.



Representation of probability content in the textbook

- ❖ Attach high importance to students' hands-on activities, experiments, and their real life experiences
 - ❧ help students learn probability through different kinds of experimental activities
 - ❧ help students experience “uncertain phenomenon”, “randomness”, “the meaning of probability” and “the relation between frequency and probability” through activities
- ❖ Make use of simulations
- ❖ Arrange the content in a spiral way



Classroom teaching

- ❖ Pay attention to students' real life experiences
- ❖ Highlight the importance of teaching in activities
- ❖ Encourage students to use computer to deal with real data

Classroom teaching (Grade 6)

Some episodes in a lesson of “the size of possibility”

❖ Introduction:

❧ How to decide who will serve the ball in a table tennis game?

❖ Hands on activity I:

❧ The teacher shows a die. Number 1 to 6 is on each of the six faces of the die respectively. The teacher asked the students to select one number from 1 to 6 which they like the most.

❧ How many times of die-tossing does it take for you to get the number you like most? (Altogether the teacher asked the students to toss six times)

Classroom teaching (Grade 6)

Some episodes in a lesson of “the size of possibility”

❖ Hands on activity II:

- ❧ The teacher shows the students some yellow balls and white balls.
- ❧ If I put the yellow and white balls in a bag and I want the possibility of taking a yellow ball from the bag to be one third, how many yellow and white balls should I put in the bag?
- ❧ Students provided different ways:
 - ❖ One yellow ball, and two white balls.
 - ❖ Three yellow ball, and six white balls.
 - ❖ As long as the number of white balls is two times that of the yellow balls.
- ❧ There are so many ways. Which way will enable us to have a better chance to get the yellow ball?
- ❧ Do experiment (group work ----- whole class discussion).

Assessment

- ❖ How to assess students' ability to calculate the probability of simple random events?

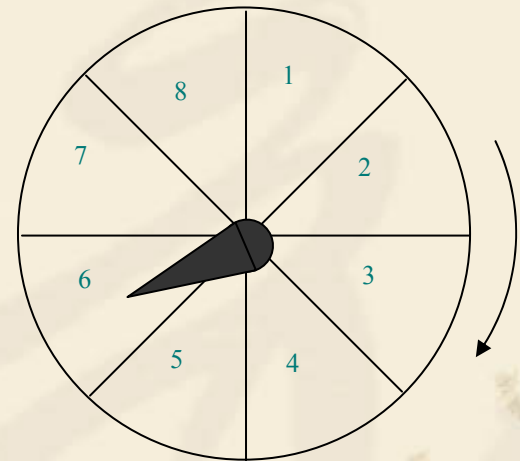
Different contexts

Different representations

☞ e.g.1. If you take any one of the six figures, i.e., an equilateral triangle, a square, an isosceles trapezoid, a regular pentagon, a rectangle, and a regular hexagon, the probability of the figure to be central symmetric and axis symmetric is _____ .

Assessment

☞ e.g.2. As shown in the figure, the spinner is divided into eight congruent sectors and each sector is labeled 1, 2, 3, 4, 5, 6, 7 and 8, respectively. Rotate the spinner. The probability of the pointer points to the number which is less than 3 is _____ .



Assessment

e.g. 3. There is an algebraic expression in each of the following three cards. Shuffle the cards and let the back side of the cards turn upward. Xiaoming closes his eyes and selects one card randomly, and then selects another card randomly from the remaining two cards. He uses the first selected card as the numerator and the secondly selected card as denominator. What is the probability of making a fraction in this way?

 x $x-1$ 2


Assessment

❖ How to assess students' understanding of frequency and probability?

✎ e.g. Xiaoying and Xiaohong did a die-tossing experiment when they were learning probability. They did 60 trials. The results are shown below.

The number shown upward	1	2	3	4	5	6
Frequency	7	9	6	8	20	10

- (1) Compute the rate frequency of “3 is upward” and “5 is upward”.
- (2) Xiaoying said, “according to the experiment, the probability of appearing 5 is the largest”; Xiaohong said, “if I toss 600 times, the frequency of ‘6 is upward’ is exactly 100”. Do you think they are correct? Why?
- (3) Xiaoying and Xiaohong tossed a die once, respectively. Using diagram or table to compute the probability of the event “the total of their numbers is the multiple of 3”.

The background of the slide is a traditional Chinese ink wash painting. It features a dark, gnarled branch of a plum tree that curves across the frame. The branch is adorned with small, delicate blossoms and buds, some of which are painted with a light pink or red wash. The background of the painting is a light, warm tone, with faint, larger-scale ink washes that suggest a misty or ethereal atmosphere. The overall style is characteristic of classical Chinese literati painting.

Thanks for your attention!

Examples (Grades 1~3)

- ❖ Which is a certain phenomenon in the following?
 - (1) It rains next Wednesday.
 - (2) Tomorrow there are people who walk.
- ❖ Use “certain”, “often”, “seldom” and “impossible” to describe the possibility of events happened in real life situation.



Examples (Grades 4~6)

- ❖ Label numbers on the six faces of a die, so as to make the possibility of the event “number 2 is upward after rolling” $\frac{1}{3}$.
- ❖ Investigate two teams and predict which team will win in the next game by providing your reasoning.



Examples (Grades 7~9)

- ❖ Rotate the spinner, and calculate the probability of the pointer points to the shaded part when it is stopped.
- ❖ Find the rate (frequency) of the event “the point of the pin touches the floor when you drop the pin” through experimentation.



Example 1

Birthday problem and Chinese zodiac problem

3. 生日相同的概率

400 个同学中，一定有 2 个同学的生日相同（可以不同年）吗？300 个同学呢？

可有人说：“50 个同学中，就很可能有 2 个同学的生日相同。”这话正确吗？

调查全班同学，看看有无 2 个同学的生日相同。



想一想

如果你们班 50 个同学中有 2 个同学的生日相同，那么能说明 50 个同学中有 2 个同学生日相同的概率是 1 吗？如果你们班没有 2 个同学生日相同，那么能说明其相应概率是 0 吗？

做一做

每个同学课外调查 10 个人的生日，从全班的调查结果中随机选取 50 个被调查人，看看他们中有没有 2 个人的生日相同。将全班同学的调查数据集中起来，设计一个方案，估计 50 个人中有 2 个人生日相同的概率。

随堂练习

1. 课外调查的 10 个人的生肖分别是什么？他们中有 2 个人的生肖相同吗？6 个人中呢？利用全班的调查数据设计一个方案，估计 6 个人中有 2 个人生肖相同的概率。

习题 6.4

1. 你几月份过生日？和同学交流，看看 6 个同学中是否有 2 个人同月过生日。开展调查，看看 6 个人中有 2 个人同一月过生日的概率大约是多少。

通过调查，我们估计了 6 个人中有 2 个人生肖相同的概率。要想使这种估计尽可能精确，就需要尽可能多地增加调查对象，而这样做既费时又费力，能不能不用调查即可估计出这一概率呢？

有人说，可以用 12 个编有号码的、大小相同的球代替 12 种不同的生肖，这样每个人的生肖都对应着一个球。6 个人中有 2 个人生肖相同，就意味着 6 个球中有 2 个球的号码相同。因此，可在口袋中放入这样的 12 个球，从中摸出 1 个球，记下它的号码，放回去；再从中摸出 1 个球，记下它的号码，放回去；……直至摸出第 6 个球，记下第 6 个号码，为一次实验。重复多次实验，即可估计 6 个人中有 2 个人生肖相同的概率。

你认为这种说法有道理吗？为什么每次摸出球后都要放

Example 1

Birthday problem and Chinese zodiac problem

回去？

上面的方法是用摸球实验代替实际调查. 类似这样的实验称为模拟实验.

议一议

除了用大小相同的 12 个球进行模拟实验外, 你还能想出其他方法吗?

事实上, 还可以用计算器产生的随机数进行模拟实验. 计算器产生随机数的过程如下: ①

1. 打开计算器.
2. 按 2ndf 7 键, 利用 ← 或 → 键选择 RANDI, 并按 = 键, 进入产生随机数的状态.
3. 按键 1 2ndf STO 1 2 ↓ , 输入所产生的随机数的范围.
4. 每按一次 = 键, 计算器就产生一个 1~12 之间的整数, 并显示在显示器的第二行.

做一做

两人组成一个小组, 利用计算器产生 1~12 之间的随机

① 不同计算器产生随机数的方法可能不同, 同学们可利用自己所使用的计算器探索产生随机数的具体步骤.

数, 并记录下来. 每产生 6 个随机数为一次实验. 每组做 10 次实验, 看看有几次实验中存在 2 个相同的整数. 将全班的数据集中起来, 估计 6 个 1~12 之间的整数中有 2 个数相同的概率.

这一结果与上一课的估计一致吗?

随堂练习

1. 用计算器模拟实验估计 50 个人中有 2 个人生日相同的概率:
两人组成一个小组, 利用计算器产生 1~366 之间的随机数, 并记录下来. 每产生 50 个随机数为一次实验. 每组做 5 次实验, 看看有几次实验中存在 2 个相同的整数. 将全班的数据集中起来, 估计 50 个 1~366 之间的整数中有 2 个数相同的概率.
2. 老师有 5 张电影票, 现在要将它们随机分给班上的 5 个同学, 为了保证公正, 你能利用计算器帮老师作出决定吗?

习题 6.5

1. 如果手头没有硬币, 那么你能用什么办法模拟掷硬币实验? 你能用计算器模拟该实验吗? 做一做, 看看结果如何.
2. 某种“15 选 5”的彩票规定: 从 1~15 这 15 个数字中选择 5 个 (可以重复), 如果其中有 2 个与所公布的中奖号码 (不妨设为 1, 2, 6, 8, 8) 相同, 即可获得四等奖. 利用计算器模拟实验估计获得四等奖的概率.

Example 2

Keyboard problem

阅读材料

电脑键盘上的字母为何不按顺序排列

电脑在今天已走进了千家万户，大大提高了人们学习与工作的效率，当你用指尖敲打着电脑键盘的时候，不知是否想过：键盘上的字母为什么不按顺序排列？

我们不妨一起来做一项统计：先选读一篇文章（见附文），然后统计总的字母数，每个字母出现的频数及频率（见附表），可以发现灵活手指操作键的区域中字母出现的频率一般较高，这样就体现出了不按字母顺序排列的优越性。

这个实验的方法并不复杂，有兴趣的话，你不妨尝试一下，多选几篇不同类型的文章，看看是否有新的发现？

前面所统计的许多实验告诉我们，实验的次数越多，就越能得到与机会的真实值更为接近的估计值。这里也一样，统计的文章越多越长，你就越能看出电脑键盘不按字母顺序排列的好处。

附文

An old man died and left his son a lot of money. But the son was a foolish young man, and he quickly spent all the money, so that soon he had nothing left. Of course, when that happened, all his friends left him. When he was quite poor and alone, he went to see Nasreddin, who was a kind, clever old man and often helped people when they had troubles. "My money has finished and my friends have gone," said the young man. "What will happen to me now?" "Don't worry, young man," answered Nasreddin. "Everything will soon be all right again. Wait, and you will soon feel much happier." The young man was very glad. "Am I going to get rich again then?" he asked Nasreddin. "No, I didn't mean that," said the old man. "I meant that you would soon get used to being poor and to having no friends."

附表:

字母总数	597															
食指管辖的字母	左	R	T	F	G	V	B	右	Y	U	H	J	N	M		
出现频数	19	40	12	17	5	4		16	34	40	0	66	38			
出现频率	3.2%	6.7%	2.0%	2.8%	0.8%	0.7%		2.7%	2.3%	6.7%	0%	11.1%	3.0%			
中指管辖的字母	左	E	D	C				右	I	K						
出现频数	66	37	5					36	3							
出现频率	11.1%	6.2%	0.8%					6.0%	0.5%							
无名指管辖的字母	左	W	S	X				右	O	L						
出现频数	38	31	0					52	29							
出现频率	3.0%	5.2%	0%					8.7%	4.9%							
小指管辖的字母	左	Q	A	Z				右	P							
出现频数	2	55	0					12								
出现频率	0.3%	9.2%	0%					2.0%								

15.3

模拟实验

1. 用替代物模拟实验

在前面的实验中我们都有现成的实物作为工具，但有时手边恰好没有相应的实物，或者用实物进行实验困难很大。这时就需要借助替代物进行模拟实验。

问题1

- (1) 在“抛一枚均匀硬币”的实验中，如果没有硬币，该怎么办？
- (2) 在“掷一颗均匀骰子”的实验中，如果没有骰子，该怎么办？
- (3) 抽屉里有尺码相同的3双黑袜子和1双白袜子，