

Some fundamental ideas in probability

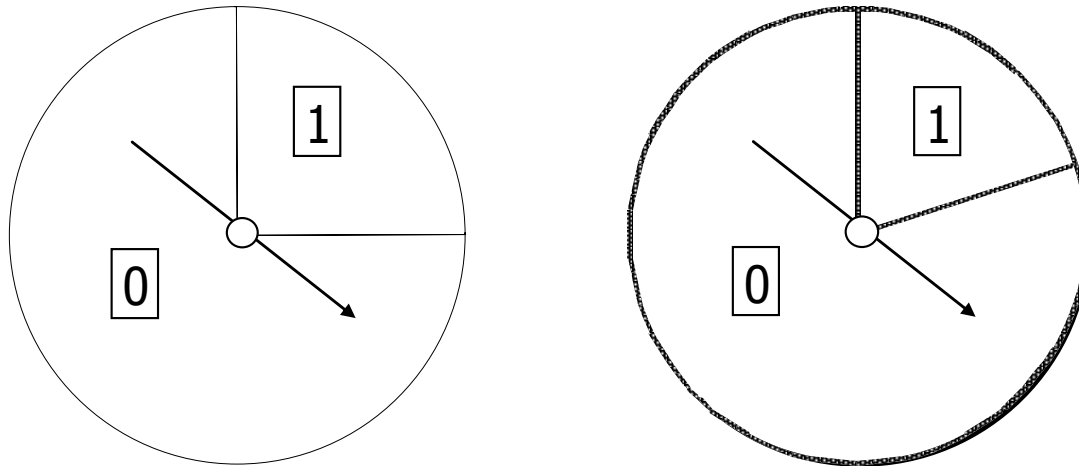
Manfred Borovcnik, Klagenfurt

Thinking probabilistically

Weighing the evidence

The paradox of stabilizing and fluctuating

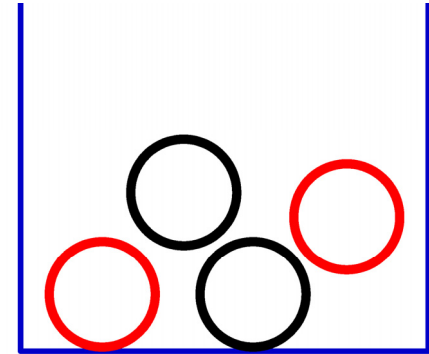
Thinking probabilistically – Falk wheels



Feedback is indirect – you may win with the wrong strategy

Thinking probabilistically – Falk Urn

- Draw twice. The drawn marble will not be replaced.
- You see the first marble, it is **red**. How big is the probability that the second marble is also **red**?
- The first marble is hidden from you and laid at the side. Then the second marble is drawn and it is **red**. What is the probability that the first is also **red**?



Interference with causal perception might lead astray

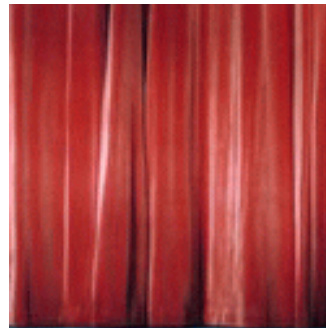
We have other criteria for decisions than optimizing random possibilities to win

Choose one of the 3 curtains (doors), the main prize is hidden behind one of them. Instead of simple guessing the candidates undergo the following procedure:

- ❑ All curtains are closed.
- ❑ The candidate chooses one, ...
- ❑ ...the moderator reveals one of the others as „goat“ ...
- ❑ ...and offers to the candidate the possibility to change the first choice



Why do people neglect mathematical results so emotionally laden?



Weighing the evidence

- The problem dealt with here is whether one should take out a policy for a comprehensive insurance of one's car for the next year.
- The focus is not on mapping the situation precisely onto a model but on illustrating matters; the rough model should just highlight the situation and the purpose of modelling by probabilities.

Cost [in Euro]		Decision	
		A_1 = Insurance yes	A_2 = no
Potential future	T_1 = No accident	1 000	0
	T_2 = Total wreckage	1 000	20 000

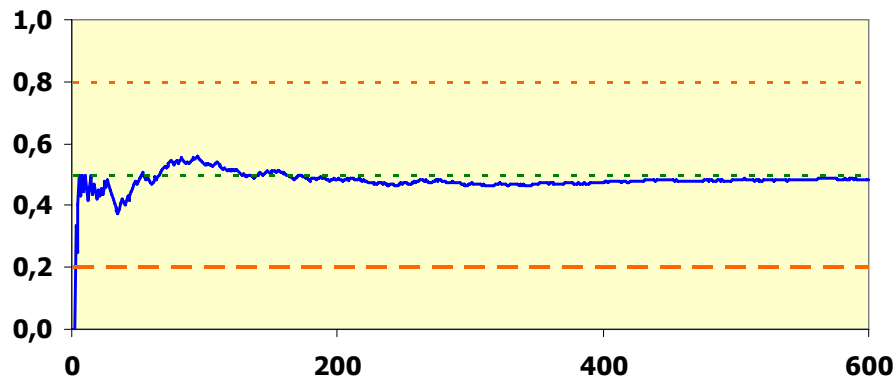
Weighing the evidence (2) – is more than frequencies

- The **single person** has to rely on **personal judgement** about his/her driving qualities and involved risk. It has also to include **utilities** of the various outcomes instead of money
- An **insurance company** includes **frequencies** of accidents and the damage in **money** and not in utilities
- The elicitation of personal odds for a total crash might be difficult and vague. It might be better to refer to an “**interval of odds**”, which seem to be plausible.
- In other problems **qualitative knowledge** of engineers might be available

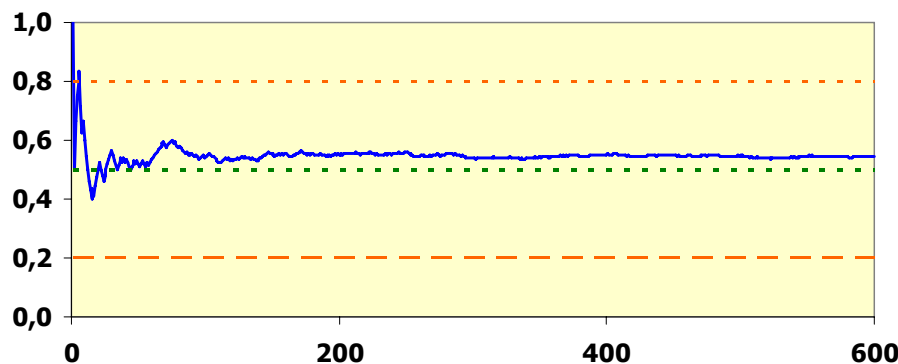
Cost [in Euro]		Decision	
		A_1 = Insurance yes	A_2 = no
Potential future	T_1 = No accident	1 000	0
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The paradox of stabilizing and fluctuating

Relative frequencies stabilize



New series stabilizes elsewhere

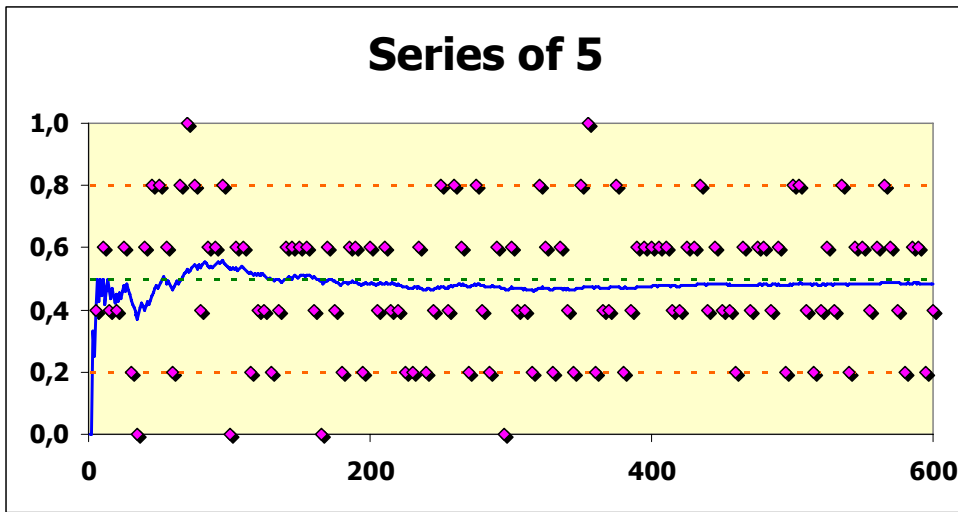


Misleading impression of the already attained precision:

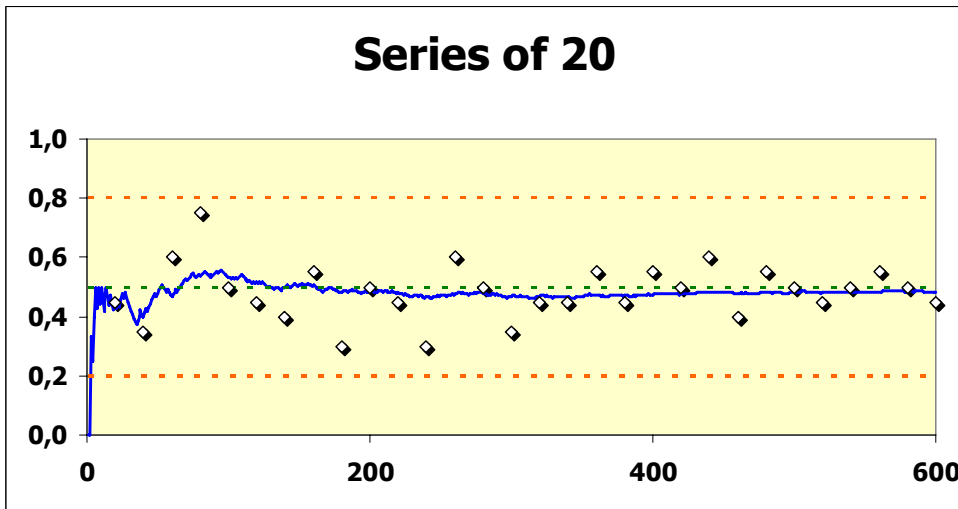
The curve suggest less than 0,5%-points variation

BUT: a new experiment would fluctuate with +/- 4%-points around the (true) p.

The paradoxon of stabilizing and fluctuating



Despite the stabilizing effect:
Single series of 5 values
continue to fluctuate
"irregularly"!



Data from the measurement
procedure based on 20 data
fluctuate much less than
data from the measurement
based on 5 data

- Randomness does not exist
 - It is only a form to think about the world
 - We have strong interrelations with other kinds of thinking, which might lead us astray
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Importance of probability teaching

- Only in rare cases results from data handling speak for themselves and allow a clear message without referring to probability
- The peculiarity of probabilistic thinking in contrast to logical, causal, or mystic thinking is important to elaborate on
- To clarify the abundance of intuitive, personal thought on probability will help to get stable intuitions about probability and its potential
- To clarify the mutual dependencies between frequentist, Bayesian, and mathematical conceptions and intuitive thought makes the concept flexible and robust
- Basic notions of expected value, risk, variability rely on sound conceptions of probability
- The historical emergence of the concepts was embedded in games of fortunes. Still they provide a source for understanding and reference models for real situations

Importance of probability teaching

- A restricted primitive notion of probability as frequency in the long run would miss other qualitative source of information for weighing the evidence, which are valuable in many cases
 - Any inferential statistical methods is intermingled with conditional probabilities and a sound understanding thereof
 - Probability is much more often used in the sense of scenarios than it is used within a model, which is aimed to depict the most relevant parts of reality – there is a need for a reference concept much wider than the frequentist approach can supply
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