

Hands-on Modelling with Wason Cards and Tinker Cubes: First steps in logical and Bayesian reasoning in fourth grade

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Historical Background

Thesis (Laplace) : Probabilistic reasoning is not just the normative framework for rational decisions but also „common sense reduced to a calculus“

Antithesis (e.g., Tversky & Kahneman): The human mind is not „hardwired“ for probabilistic reasoning

Synthesis: The modern view in cognitive psychology is that humans' capability to reason probabilistically often depends on the representation of information (e.g., Gigerenzer)

Some examples ...

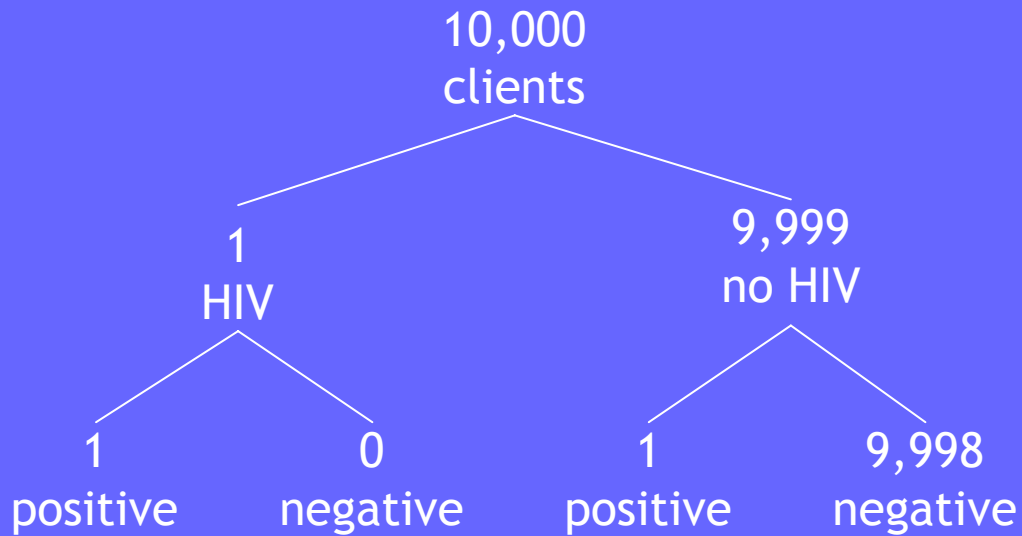
- 1) Bayesian Reasoning (HIV-Test)
- 2) Conjunction fallacy (Linda Task)

All examples are known as famous cognitive illusions.

However, „natural“ frequencies can help in each case ...

- 3) Can the natural frequency approach even be applied with 4th graders?

1) Natural Frequencies



$$p(\text{HIV} \mid \text{positive})$$

$$= \frac{1}{1 + 1}$$



Conditional Probabilities

$$p(\text{HIV}) = 0.01\%$$

$$p(\text{positive} \mid \text{HIV}) = 99.9\%$$

$$p(\text{negative} \mid \text{no HIV}) = 99.99\%$$

$$p(\text{HIV} \mid \text{positive})$$

$$= \frac{.0001 \times .999}{.0001 \times .999 + .9999 \times .0001}$$



2) Conjunction Fallacy

Linda is 31 years old. She majored in philosophy and was active in the anti-nuclear movement. When Linda was in highschool she was outspoken.

Which statement is more likely to be true:

- a) Linda is a bank teller
- b) Linda is a bank teller and a feminist

Making cognitive biases disappear with frequencies...

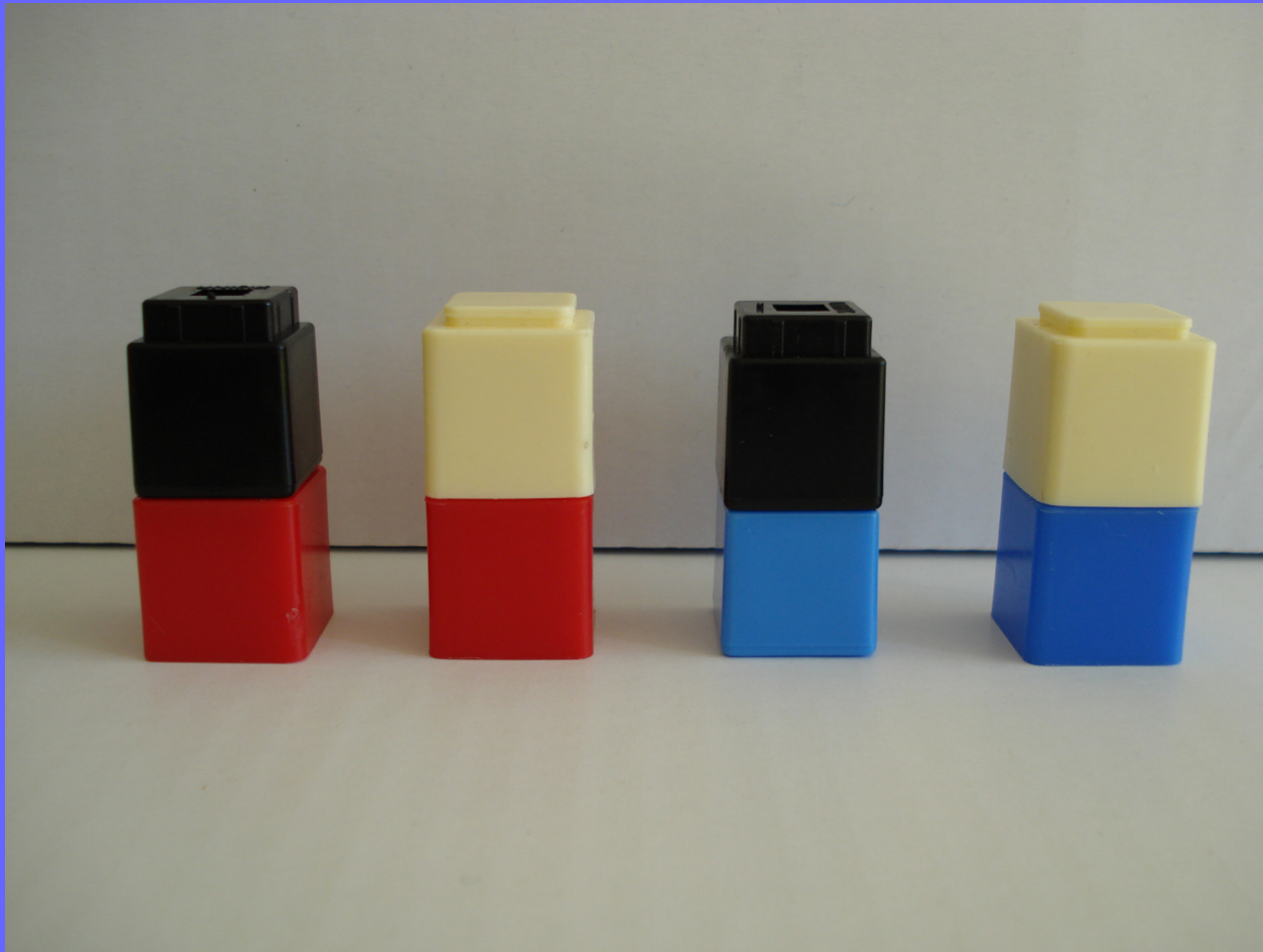
Linda is 31 years old. She majored in philosophy and was active in the anti-nuclear movement. When Linda was in highschool she was outspoken. Imagine 100 women that satisfy this description.

How many of them are

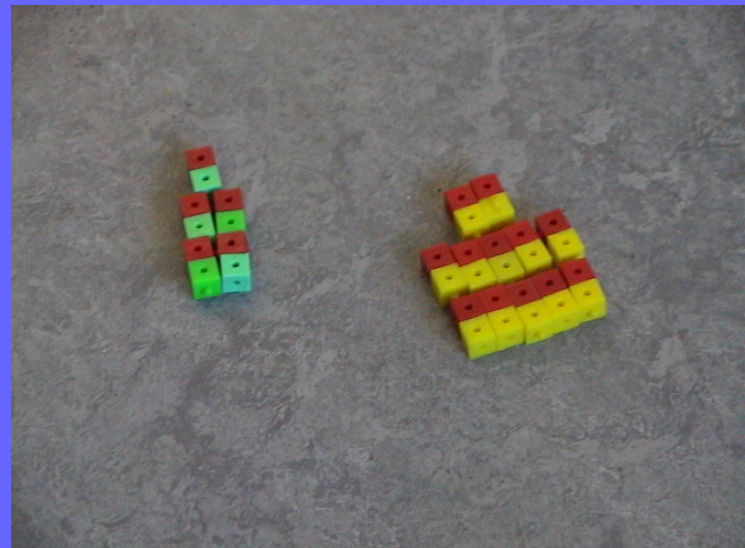
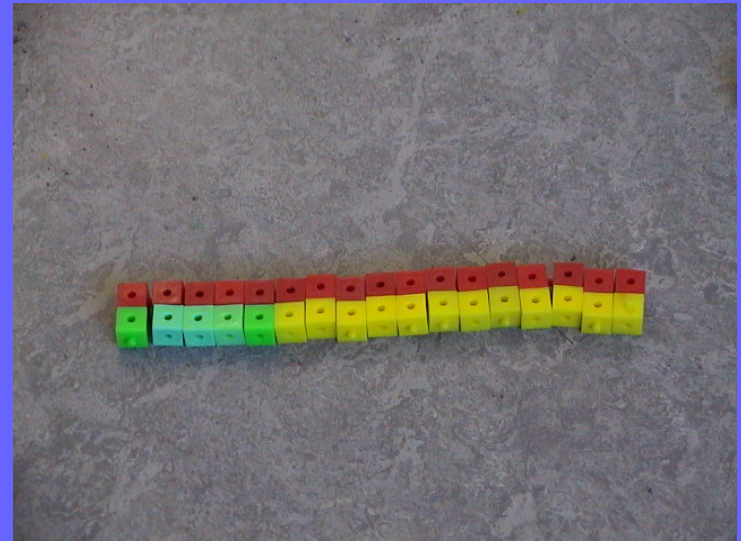
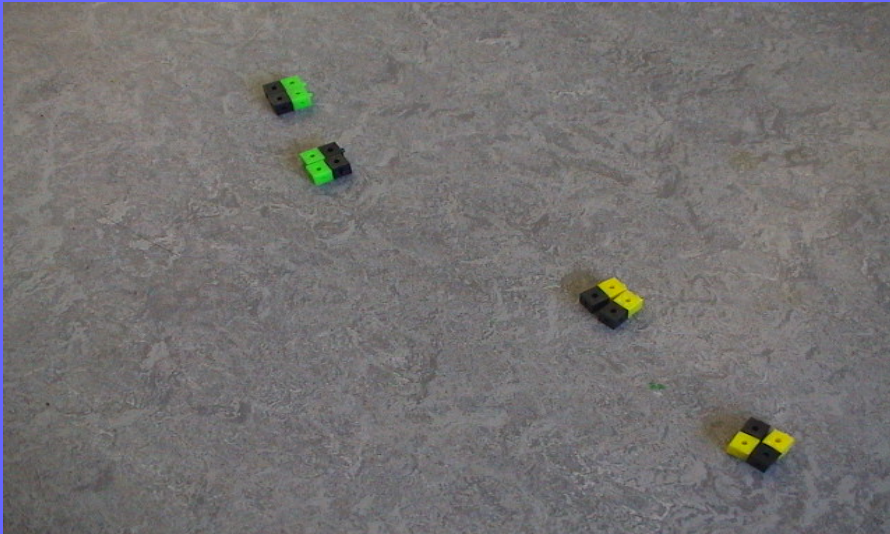
- a) Bank tellers
- b) Bank tellers and feminists

3) Can 4th graders be introduced to the basic ideas of Bayesian Reasoning?

Hands-on analogue modeling with Tinker Cubes

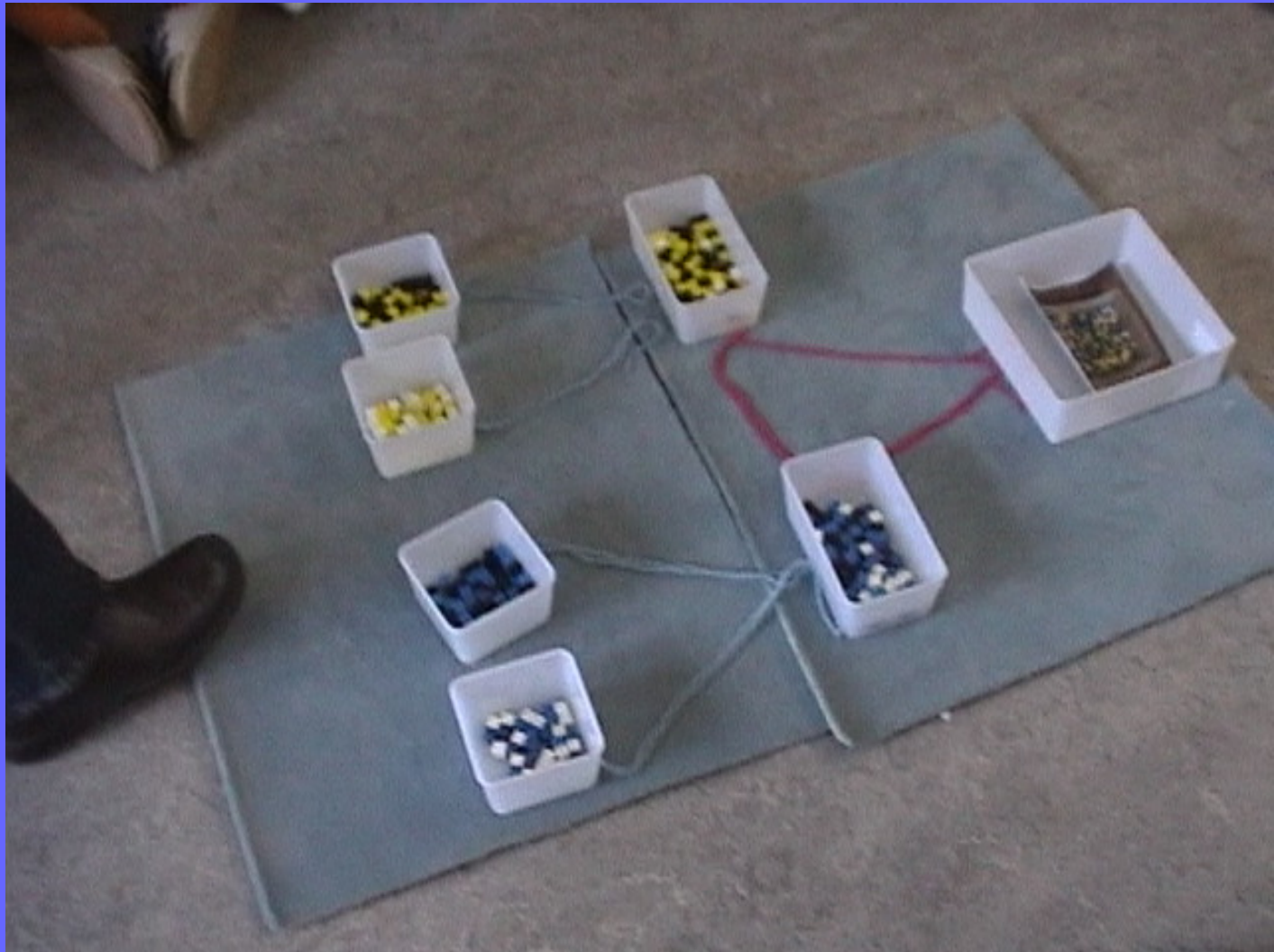


The boys and the girls





Lively urns and eventful trees

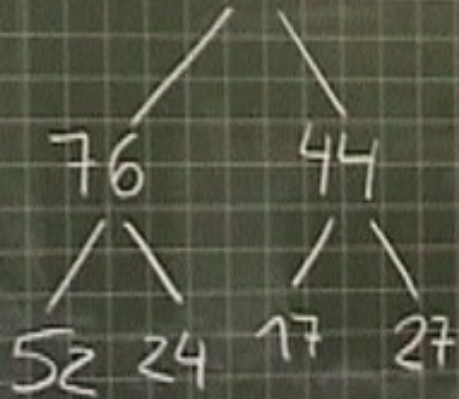


77 44
27

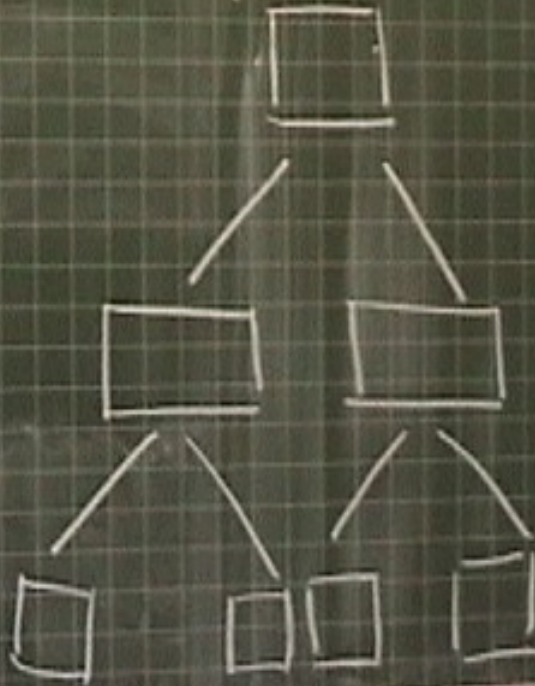
120

24 76
52

Kino
120

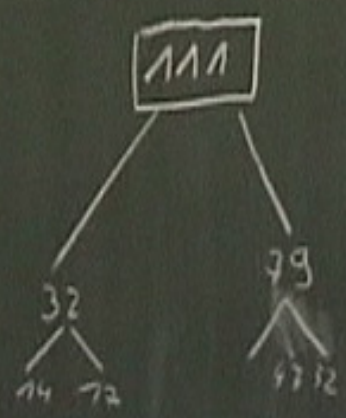
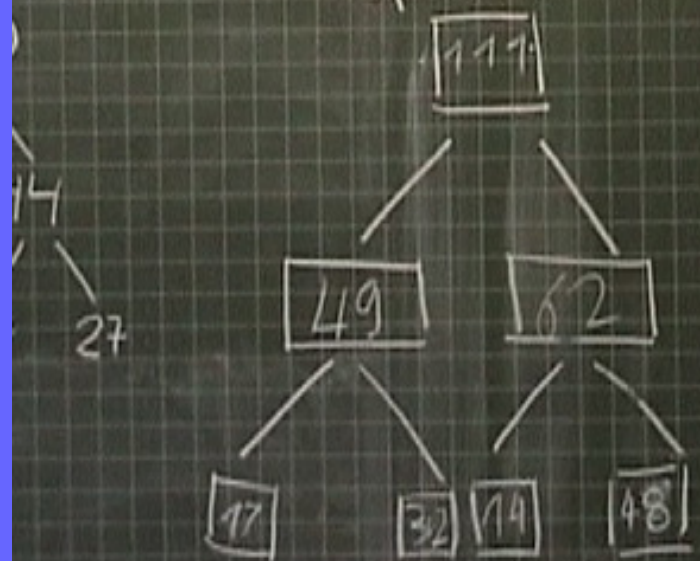


Fußball



tauschen, plaudern, a-
miden, abhören, and-
worten, erwidern,
entsagen, verlangen

Fußball



References:

Empirical results: Kurz 2008, Martignon, (JRME, in revision)

Martignon & Kurz-Milcke, 2007

Martignon & Krauss, 2008

Kurz-Milcke, Gigerenzer & Martignon, NYAS 2008