

Probability Teaching in Brazilian Basic Education: Evaluation and Intervention

Verônica Yumi Kataoka

Ademária Aparecida de Souza

Anderson de Castro Soares de Oliveira

Fabírcia de Matos O. Fernandes

Patrícia Ferreira Paranaíba

Marcelo Silva de Oliveira

Federal University of Lavras (UFLA) and University Bandeirantes (UNIBAN)

veronicayumi@terra.com.br

Summary

In this article we develop a scenario of probability teaching in Brazil: curricular issues, teacher formation, didactic books, and activities developed by researchers. Next, we present the following inquiry: which pedagogical actions may contribute to an effective improvement in probability teaching? From our research, we conclude that, in the short time, such actions may include: investing in the continuing process of teachers' formation, and proposing pedagogical activities which allow the pupils to develop probabilistic reasoning, and then to formalize concepts. According to these ideas, we have designed special workshops for teacher in-service. Each of the workshops for a group of basic mathematics education teachers was carried out in three meetings of 4 hours each, in which we worked on several topics in probability and statistics. In this paper, we describe the last meeting with the title "Influence of previous knowledge in a Bayesian approach", in which we dealt with the following concepts: prior information; concepts of probability like classic, frequentist, geometric and conditional probability, and the Bayes' theorem. Beyond the positive results reached in this workshop, we detected the need of planning and elaborating more didactical sequences that allow the pupils to improve their probabilistic reasoning, and to follow-up these teachers' work at school in order to verify pupils' reactions to those kinds of activities, and therefore, making possible an effective improvement in probability teaching in basic education.

KEYWORDS: Probability teaching, continued formation, sequence didactic, probability reasoning.

Introduction

Basic knowledge of probability is essentially important in the formation of citizens, as it allows them to understand many random facts that happen in their daily lives, in addition to acting together with statistics (data handling) in the inferential context for making decisions. However, not all probability concepts are easily understood at first, as many of them are rather abstract, requiring caution and skill from the teacher in the conduction of the teaching - learning process, in order to provide pupils with the proper development of their probability reasoning.

According to Coutinho (2001); Batanero and Godino (2002), the construction of probability concepts should start from the understanding of three basic notions: perception of chance; idea of random experience; and notion of probability. Likewise, Lopes (2003), Kataoka, Rodrigues and Oliveira (2007) state that it is desirable that the teacher would approach such concepts through activities in which pupils make experiences and observe the events, promoting their intuitive expression of chance and uncertainty, constructing, from the results, mathematical methods for the study of those phenomena. Such guidelines follow the study line that Shaughnessy (1977) calls activity-based.

In Brazil, the National Curricular Parameters (PCN) would recommend that probability be taught right from initial Elementary School and resumed in a progressive approach at every new cycle (Brasil, 1997, 1998). However, according to Lopes (2003), unfortunately the inclusion of statistics and probability into Basic Education Mathematics curricula was not followed by a careful qualification of teachers.

Dias (2004), Serrado, Azcárate and Cardeñoso (2006) mention two pedagogical difficulties for probability teaching by mathematics teachers in Basic Education: the first refers to the innovation represented by the insertion of the subject in the curriculum, requiring that teachers overcome habits and seek new information and activities to develop in the classroom. The second condition, also reported by Batanero, Godino and Roa (2004), Pecky and Gould (2005), or Ainley and Monteiro (2008), is that educators coming from mathematics graduation sometimes have some basic formation in probability and statistics but are not always prepared to teach such contents. Another aggravating circumstance in probability teaching, according to Batanero et al. (2004), is that the textbooks sometimes present a too-narrow view of probability (only the classical approach), and applications are at other times restricted to games of chance and in some of them the definitions of the concepts are incorrect.

In view of these facts, we present the following question: what pedagogical actions would contribute to an increased effectiveness in probability teaching? We believe that, in the short run, such actions would be: investment in the process of continued education of teachers and the proposition of detailed pedagogical-modeled activities which would, at first, allow pupils to develop probabilistic reasoning skills, and then formalize the concepts. In fact, our opinion is theoretically based on an analysis of articles from periodicals and summaries of recent congresses held in the area of mathematics and statistics education, in which we verify that such activities have been globally proposed by a number of educators, such as for instance: all the already mentioned references, as well as Watson (1998); Cordani (2002); Godino, Cañizares and Diaz (2003); and Lopes (2006).

Therefore, this article aims at contributing to those studies by reporting a field-work within such an approach towards teaching. In some sense, the work with teachers and the development of specific activities puts such international approaches to a practical test and will lead to refined proposals for teaching sequences and to a better know-how of organizing teacher in-service training. As a by product, the background of the local teachers could be improved by our activities and the initiation of local networks may overcome the deficits of the teachers' didactic and scientific education in probability at university.

Probability Teaching in Brazil

Before describing our experience, we will introduce a brief scenario of probability teaching in Brazil: approaching curricular matters, teacher formation, didactic books, and activities under development by national researchers for education improvement and/or enhancement.

Basic Education

In Brazil, Basic Education (or Basic School) is the designation for the learning levels corresponding to the first years of formal education and comprises Nursery, Elementary and Secondary Schools, with the ideal length of eighteen years.

The Nursery School serves pupils aged 0-5; the Elementary School is divided in two cycles: the first for pupils aged 6-10, and the second for 11-14 (in this work, this level will be referred to as Elementary II); and the Secondary School is for pupils aged 15-18. Compared to the classification as adopted in other countries,

Nursery and first cycle are usually named Elementary School, the Elementary II School corresponds to Middle School, and the Secondary School is High School.

National Curricular Parameters

In most countries, the insertion of probability and statistics in mathematics curricula is relatively new. In addition, there are differences among those curricula, with some of them rendering added importance to mathematical aspects of statistics, while others emphasize data analysis and interpretation, and there are also curricula, which consider statistics as a language to describe and interpret the reality (Ponte & Fonseca, 2001).

In Brazil, according to Gonçalves (2004), the first institutional guidelines involving probability teaching in curricula date back to the 1980's, and they are completely restricted to the Secondary School. Only after 1997, with the publication of the PCN – the National Curricular Parameters for the Elementary School (Brasil, 1997), the inclusion of probability and statistics into curricula was effective, with consistent changes in the National Education scenario regarding probability and statistics teaching.

The PCN has been prepared, on the one hand, trying to respect regional, cultural and political diversities existing in the country, and, on the other hand, considering the need to build common nation-wide references in education for all Brazilian regions. The Mathematics PCN aimed at building a reference to guide the school practice so as to contribute for every Brazilian child to have access to mathematical knowledge allowing them to be inserted, as citizens, in the world of labor, in social relations, and in culture (Brasil, 1998).

Mathematics PCN consist of content blocks; and suggest that probability and statistics topics be viewed as part of a block known as “Treatment of Information” (data handling) in the first (Brasil, 1997) and second cycles of Elementary School (Brasil, 1998), and in the “Data Analysis” block for the Secondary School (Brasil, 2002, 2006), in the mathematics subject.

In the Treatment of Information block, PCN indicates that the objectives of probability teaching for the first cycle of Elementary School are:

[...] to enable pupils to understand that a great deal of daily occurrences is random by nature, and it is possible to identify probable outcomes from such occurrences. The intuitively acquired notions of chance and uncertainty can be further explored at school, in situations where the pupil makes experiments and observes events (in equiprobable spaces) (Brasil, 1997, p. 40)

[...] to enable pupils to use the given information to calculate probabilities. To identify possible ways to combine elements forming a collection and take them into account using personal strategies. (Brasil, 1997, p. 62)

For Elementary II School, PCN indicates the following objectives:

[...] to enable pupils to represent (tables, tree diagram) and report possible cases in combinatorial situations; describe sample spaces in situations such as throwing dice and tossing coins, and determine the possibility of success of an event by giving reasons (Brasil, 1998, p. 75 and 77).

[...] to enable pupils to describe sample spaces by using the multiplicative principle. and the indication of probability of an event a ratio of the numbers of favorable and possible events. To prepare experiences and simulations that would estimate probabilities and verify anticipated probabilities (Brasil, 1998, p. 90).

In the “Data Analysis” block, in turn, the probability teaching objectives are:

[...] to enable pupils to understand that probability is a measure of uncertainty, that models are useful to simulate events, to estimate probabilities, and that sometimes our intuitions are incorrect and can lead us to erroneous conclusions about probability and chance.

[...] to enable pupils to learn and describe random situations and experiences in terms of possibilities, associate them to a set of elemental events and represent them in a schematic way (two-way table and tree diagram).

[...] to enable pupils to master the language of events, to formulate equiprobable hypotheses, to associate the probability of observed results with the frequency of corresponding events, and use the statistics of such frequencies to calculate the probability of a given event – conditional probability (Brasil, 2006, p. 79-80).

When the PCN recommendations are compared to mathematics curricula of other countries, for instance, England (DFEE, 1999) and USA (NCTM, 2000)¹, both concordant and discordant points are observed for probability teaching.

In the case of the British mathematics curriculum at level KS 2 (equivalent to the first cycle of Elementary School), pupils should understand the concept of probability and discuss events by using terms such as equally probable, unfair, and fair; at levels KS 3 and 4 (corresponding to Elementary II stage), the pupils should understand the concepts and properties of probability, identify results of simple events, or of two successive events, as well as mutually exclusive events, and the observation of the convergence phenomenon. At the advanced KS 4 (equivalent to Secondary School), the understanding and calculation of probability of independent events is added (DFEE, 1999; Ponte & Fonseca, 2001).

Ainley and Monteiro (2008) developed a comparative study between Brazilian and British curricula for the Primary School, focusing on the teaching of graphical construction, and one of their findings was that “... both curricula suggest that data handling should be approached as a problem solving process and connected with pupil’s daily contexts but do not give details about how teachers could achieve such challenging aims”.

In the case of the USA, the NCTM suggests that Elementary School pupils should study probability first in an informal way (with notions such as more probable, less probable, certain or impossible), and, next, quantify the probability of simple events. At Middle School, pupils should understand and use the proper terminology to describe complementary and mutually exclusive events, using probability calculation for composite events using tables and tree diagrams, in addition to encouraging the use of experiences and simulations. At High School, pupils should calculate probabilities of composite events, including independent events and conditional probabilities, in addition to using simulations to build probability distributions for describing simple sample spaces (NCTM, 2000; Ponte & Fonseca, 2001).

In summary, as examples of common and discordant points of recommendations we highlight:

- Notions of chance, uncertainty and possibility – all three documents recommend these notions in the first Elementary cycle;
- Probability of a simple event in a informal way – PCN and the U.S. document suggest it in the first cycle Elementary School; the British only at the Elementary II stage;
- Use of tables and tree diagram, combinatory analysis for the calculation of probabilities are recommended only by PCN and the U.S. document for Elementary II School;
- Independence of events – all three documents recommend this concept only for the Secondary School;

¹ In the USA there is no national curriculum, therefore the reference is the document called Principles and Standards for School Mathematics (NCTM, 2000).

- Conditional Probability – only PCN and the U.S. document recommend dealing with this concept, and only for the Secondary School.

The practical implementation of the PCN recommendations for the approach of the subject in the classroom is under the responsibility of mathematics teachers, as there are no graduation courses in statistics, statistics is not even integrated as a subject in Basic Education programs.

Teacher Formation

In the structure of the Basic Education, the formation of mathematics teachers differs depending on the school level they will teach. In the case of the first cycle of Elementary School, teachers graduate in “Teaching Higher Education” or “Pedagogy”, and in the case of Elementary II and Secondary School, teachers graduate in “Mathematics” (Figure 1).

Fernandes, Ferreira, Kataoka, Souza, Resende (2008) observe in their studies that currently in Brazil, over 75% of all Federal Universities (UFE) offer a major in mathematics. Only two UFEs, considering only those with information in an official website (approx. 65%), do not include a probability and statistics related subject in their curricular grid. On average, the programs offer only one mandatory subject. (In fact, only an analysis of the study synopses, which is not offered by all universities, would allow us to verify whether the contents are really taught or not in practice.)

Another finding in the study by Fernandes et al. (2008) is that, from the available information, the absence of subjects related to probability and statistics in major mathematics courses is a serious deficiency in the initial formation of teachers. Maybe it derives from the fact that the curricular guidelines for mathematics courses by the Ministry of Education (Brasil, 2002), require the insertion of the probability and statistics only in graduate mathematics, and it is not included in the basic structure of the major in mathematics program.

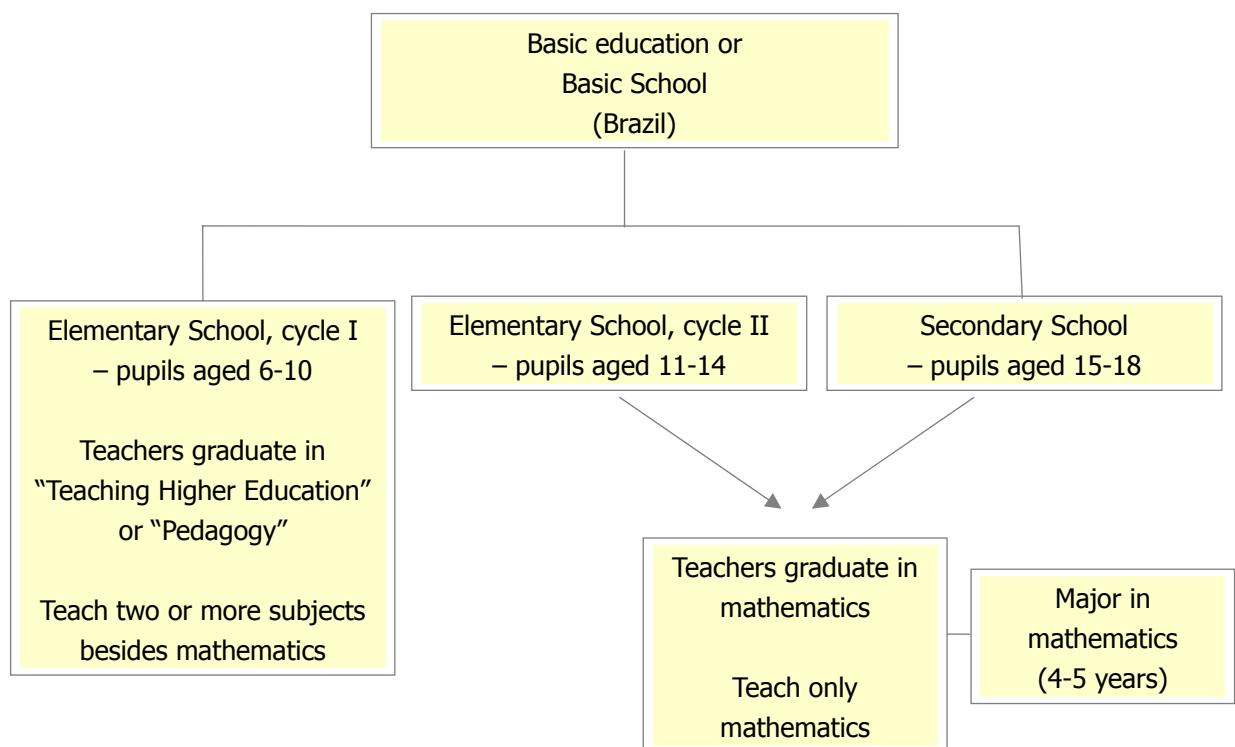


Figure 1. Schematic chart for the formation of mathematics teachers according to the school level of Basic Education in Brazil.

In the curricular grid, there is no subject related to aspects of teaching didactics in probability and statistics, not even as an elective subject (Fernandes et al, 2008). Cazorla (2006) also reinforces that the same happens when analyzing the curricular form of the courses in areas related to statistics. To be familiar with the mathematics of probability and statistics, is a prerequisite for teaching, but teachers have also to know how to teach its elements and if they are not able to teach the contents clearly, the learning process may be doomed to failure.

Didactic Books

Although probability has been included as compulsory element into curricula only recently, the subject has been addressed by didactic books already over a longer period. Lopes and Moran (1999) studied some didactical books recommended for the Elementary School, and pointed out that in introducing the concept of chance the word "probability" is not used, and the concept of probability is defined in the Classic way, as the number of favorable cases divided by the number of possible cases, without mentioning the restriction of equiprobability and the set of possible cases being finite. Even more, according to these authors, if any student, working with just that definition, would try to calculate the probability of the weight of a pupil to be between 50kg and 55kg he could not complete the task, since the set of weights is infinite and incommensurable. So, the recommendation would be that the frequentist definition, numbers of occurrences of the event divided by the number of repetitions, should be given, since it will allow dealing with probability even if both restrictions are not fulfilled.

In his endeavor to establish a scenario of probability teaching across the curriculum, Gonçalves (2004) analyzed some didactic books from the 1970's, 80's and 90's and concluded that probability has been taught by classic and axiomatic approaches, and there has been variation only in the types of tasks, techniques, and theoretical-technological discourses of the proposed situations introduced as exercises or examples. That is, in the 1970's, the techniques to present and solve the tasks were based on set theory, in the 1990's on combinatorial analysis, while the 1980's represent a transition period in which both theories were used to justify the techniques in use.

Coutinho, Lopes and Cordani (2004) also prepared a study on Brazilian didactic books intended for the Elementary II School, and verified that there were no suggestions for work within the experimental approach towards probability, which would help to develop the frequency point of view in the concept of probability.

Oliveira (2006) surveyed ten didactic books adopted or indicated by a large number of mathematics teachers in the secondary school. The survey found a wide variety of symbols and formulas and concepts used in probability-related contents, sometimes too complex for secondary school pupils, as they do not present a coherent body of theory and they appear even more complicated due to the contexts in which they are used, as the presented examples are not significant, and only a few are relevant inform an interdisciplinary aspect.

Friolani (2007), in turn, found a series of books proposing activities from classic and frequency approaches, as well as activities that try to establish an interaction between the concepts of probability and the concepts of statistics. It is worth mentioning that the teacher should be careful about the frequency approach adopted by some books, as experiments in the classroom might not produce the desired effects due to the "sensibility of the sample size", according to Batanero (2001, p. 104) this happens because of a wrong extension of the "Law of Large Numbers", supposing the existence of a "Law of Small Numbers". According to Kataoka, Cazorla, Silva and Oliveira (2008), such a belief might lead to unjustified expectations in the replication of random experiments performed with small samples, therefore compromising the teacher's

work and making it difficult for pupils to understand. As well stated by Fischbein (1987), human experience is necessarily limited in time, space, and set of possibilities.

Actions

Currently, in Brazil, large efforts are made aiming at improving knowledge and professional development of mathematics teachers who deal with probability and statistics; among these endeavors we highlight:

- Activities of continued education – development of pedagogical workshops and mini-courses intended to promote a continued formation in probability and statistics with mathematics teachers, some works already developed include Cordani (2002), Lopes (2006), Kataoka, Oliveira, Rodrigues, Souza and Oliveira (2007).

In addition to the activities promoted by single local groups, the Ministry of Education (MEC) created the National Network for the Continued Education of Basic School Teachers with the aim to contribute to sustainably updating the formation of teachers and pupils. This network is based on *Centers for Research and Development in Education*, which are situated at universities; in the area of mathematical and scientific education the universities involved are: the Federal University of Pará – UFPA, the Federal University of Rio de Janeiro – UFRJ, the Federal University of Espírito Santo – UFES, the State University of Sao Paulo – UNESP, and the University of Vale do Rio dos Sinos – UNISINOS. Some projects in the field of probability are already under development at UNESP and UFPA.

- Congresses and meetings – with incentives by some research groups, some congresses and meetings have been held approaching the topic of probability teaching, such as the “National Meeting of Mathematical Education”, in addition to regional meetings; within the “Brazilian Society of Mathematical Education” a group on Statistics and Probability Research (GT12) was established.
- Theses and dissertations – In post-graduate mathematical education, some works have been developed in probability and statistics teaching within programs such as PUCSP, Unicamp, UFRGS, Unicsul and UFPA.
Coutinho (2008) presented a synthesis of dissertations and theses defended at PUCSP in the period 2006-2007 related to a research group called PEA-ESTAT (Statistics Teaching and Learning Processes).
- Publication – articles in magazines such as “*Professor de Matemática*”, “*Educação Matemática em Revista (EMR)*”. As examples of works published in EMR, we mention: Echeveste, Bayer, Bittencourt and Rocha (2006); Grácio and Garrutti (2006); Gonçalves and Muniz (2006); Oliveira and Cazorla (2008); Ângelo and Silva (2008); or Pasqualotti and Koehler (2008). All mentioned works approach aspects related to probability and/or statistics teaching.
- Publication of the book “Treatment of Information for Fundamental and Secondary Schools” by Cazorla and Santana (2006). This book presents some detailed orientations for the development in the classroom of four didactical sequences are devoted to topics on statistics and probability.

In this context of collaborative effort of the researchers for upgrading statistics and probability teaching, we are going to describe our experience with a workshop for mathematics teachers, particularly about the sessions dealing with probability concepts.

Pedagogical Workshop – General Description

Selection and Profile of Teachers

For the selection of teachers in this study, we initially selected by chance 30% of the schools in the municipality of Lavras, with equal proportion of public and private schools. Next, through a voluntary enrollment process, 30 teachers were selected, being 21 from Elementary II, and 9 from Secondary, corresponding to 25% of the total of mathematics teachers in the city. The selection was not at random because teachers depended on schedule availability to confirm their attendance to the three meetings proposed by the workshop; nevertheless we believe that those teachers' experience in the workshop is a valuable source of information worth to report and back up further practical research in improving teachers' abilities and knowledge for teaching probability and statistics.

Inquiry

After the selection process and before setting up the workshops, the teachers answered a questionnaire consisting of 21 questions which aimed at providing an overview of training, professional experience, and personal background, as well as factors regarding the teaching of probability and statistics such as, for instance, the content that was dealt with and the kinds of didactic resources that were employed.

With regard to their professional background, it can be noted that all teachers have graduated in mathematics, have an average professional experience of 14 years, and a minimum workload of 20 hours per week. Most of the teachers stated that, in their major, they had only studied one subject-area that involved concepts of probability and statistics but had not been given any detailed guidance on questions regarding the teaching of those concepts; these results corroborate the findings of the research carried out by Fernandes et al (2008). In the opinion of those teachers, this lack of preparation on how to teach probability and statistics during their formation has constituted one of their main difficulties in combining theory and practice in the classroom, in their early stages as teachers.

Regarding the contents taught in the classroom, teachers in the Elementary II School stated that probability is restricted to cases with equiprobability and the calculation of the chance of simple events without combinatorial tools. Teachers in secondary school use combinatorial analysis as a principal means of approaching probabilistic concepts and deal with the probability of simple events, the properties of probability, and the concepts of conditional probability and independence of events. However, in general, those matters are only taught in a mechanical way through the manipulation of formulas without any kind of contextualization, that is, only by reproducing the concepts and exercises in exactly the same way in which they are treated in the didactic books. According to the teachers, those didactic books are often found to be the only teaching material available in the school, since most schools do not have a computer laboratory or audiovisual equipment.

Moreover, most teachers expressed their need to improve their expertise in teaching probability and statistics getting involved in other interdisciplinary areas where their teaching activities can be put into effect. This need on the part of the teachers has been a motivating factor in encouraging us to set up a pedagogical workshop.

Sessions

Two pedagogical workshops were conducted in the Department of Exact Sciences at the Federal University of Lavras. One of these was for mathematics teachers at Elementary II School and the other for Secondary School teachers who are currently teaching in both public and private schools in Lavras. The two workshops were carried out on the basis of the same pedagogical model and were only distinguished by the

depth of some of the explored concepts and the sequence of the topics being taught. These different ways in conducting the workshops were not caused by the fact that the teachers came from different backgrounds but arose only from the different goals of the curricula in teaching probability for the two academic levels.

Each of the workshops was divided into three sessions lasting four hours each. They were partly conducted with a view to strengthening theoretical areas but were mainly designed to carry out various activities that had a practical character which were called didactic sequences. The didactic sequences were intended simultaneously to provide deeper knowledge, incentives for integration between the participants, and exemplary demonstrations that the same sequences are suitable for teaching the concepts involved to pupils. Problem-situations were proposed to challenge the teachers (either individually or collectively) to extrapolate a possible solution from a context that was purely descriptive, but which always had an inferential character.

The following topics and contents were dealt with in the sessions:

- 1st session – Exploring estimators – sampling techniques, descriptive statistics, and point estimation.
- 2nd session – Making right decisions: a statistical problem – basic notions of interval estimation and tests of hypotheses.
- 3rd session – Influence of the previous knowledge in a Bayesian approach – Prior information; Types of probability: classical, frequentist and geometrical probability, conditional probability, as well as Bayes' theorem.

This article is providing a detailed account only on the 3rd session due to the fact that it dealt specifically with issues of teaching of probability.

Before beginning the first session, a survey was conducted (Annex A), consisting of three expository questions. Each question corresponds to each one of the topics of the three meetings, the aim being to assess the level of knowledge and degree of significance of the topics for the teachers. In outlining the third question which is linked to the 3rd session, Table 1 below shows statistics from the State Highway Dept., providing information about the number of accidents with victims (fatal or otherwise), and the condition of the driver involved (drunk or sober)

Table 1. Victims of accidents by state of the driver – data from the State Highway Dept.

Driver/ victim	non-fatal	fatal
Sober	1228	275
Drunk	239	76

And we proposed the following question:

Would you say that the condition of the driver – drunk or sober – affects the occurrence of fatalities?

We expected that teachers would calculate the conditional probability (in an informal way) and would conclude that intoxicated drivers were more likely to be involved in fatal accidents. But, in fact, only eleven teachers (approx. 37%) gave the correct answer, and only four of them presented their calculations, while seven teachers only justified their answers in words. Out of the teachers who responded incorrectly, some based their wrong answer on the following calculation:

$$P[\text{Fatal} \cap \text{Sober}] = 275/1818 = 0.15 \quad \text{and} \quad P[\text{Fatal} \cap \text{Drunk}] = 76/1818 = 0.04$$

Therefore, they concluded that there was a greater probability of fatalities when the driver was sober. This kind of confusion between conditional probability - $P(A|B)$ - and the probability of intersection - $P(A \cap B)$ - was also verified in a study by Estrada and Díaz (2006), Figueiredo (2000).

The results of the pre-test served as a diagnosis, as well as a guide for the theoretical discussions during the session.

Pedagogical Workshop – 3rd Session:

Influence of Previous Knowledge in a Bayesian Approach

Initially, there was a discussion on whether prior information or previously acquired knowledge may be of help at the time someone makes a determined statement. That is, it was asked whether the experience and information conveyed to someone may enable that person to reach at quantitative judgments about the possibility of an event occurring, even in a rather subjective way. A number of questions were raised to the teachers including:

For a coffee crop planted in southern Minas Gerais, Brazil, what is the chance of a successful yield?
And how about success of the same crop planted in Bangladesh in center-southern Asia?

Without hesitation, the teachers expressed a strong likelihood that the crop would be successful in southern Minas, but that they could not provide an immediate reply regarding Bangladesh because they had no information on coffee production in that country. We should mention that coffee crops are an important local matter in Lavras and the local media would report almost daily on crop yield estimation and coffee wholesale prices. In addition, at school, pupils learn about the particular climate and soil types of the region and that Southern Minas is very suitable for coffee cultivation.

The teachers thus realized that their previous knowledge of coffee production in southern Minas had influenced their responses, although they did not know how to link this information to a quantitative value (probability estimation).

Teachers expressed that it could well be that the production of coffee might be more prosperous in Bangladesh than in south Minas Gerais as the general conditions might be better for plantation, but they stated that this was something they knew nothing about. Then, it was discussed that some information should be named as prior information only when it influences the estimate of some probability in which we may be interested. For example, if teachers knew that a coffee pest infested plantations in south Minas Gerais, this would probably lead them to a change of mind in their original opinion. On the other hand, that very same information could not be of any use in the context of Bangladesh, since teachers did not even have any certainty that coffee is produced there.

After that stage, five didactic activities were set up to apply the concepts of the 3rd session. We should stress that our main concern was not to present questions of a great theoretical complexity in any of the didactic sequences, and not even to observe only numerical results; we focused primarily on the solving strategies that were employed by the teachers.

The methodological procedures adopted to carry out the didactic activities follow guidelines laid down in the work by Cordani (2001), Lopes (2003, 2004), Mendes and Brumati (2003) and Coutinho et al (2004).

The procedure in the didactic activities may be described in the following way:

- First discussing the problems requiring a resolution from the teachers;
- Assessing the solving strategies (not necessarily using formulas) developed by them by a group discussion;
- Formalizing the concepts, also by holding a group discussion;
- Encouraging the teachers to engage in discussions such as the possibility of applying the activity in class, the importance of suitability in selecting tasks for each school grade, as well as exploring the idea of embedding the task in an interdisciplinary context.

First Didactic Sequence

For the first didactic sequence, we used data from Elementary School II for constructing the two-way table below (Table 2):

Table 2. Elementary II teachers by gender and school system.

School	Gender	
	Female	Male
Public	11	3
Private	3	4

Based on the data from Table 2, we asked teachers to calculate the probabilities of a randomly selected teacher to be

- a) male;
- b) from the public network of schools;
- c) female and from a public school;
- d) male or from a private school;
- e) from the public network of schools provided the person is female.

It was observed that in seeking their answers, most teachers (63%) took data directly from the table without using formulas, such as, for instance, for the probability of combined statements (questions c and d), or for the conditional probability (question e).

Whether with or without formulas, in answering item “e”, several teachers confused conditional probability with intersection probability, a phenomenon, which also occurred in the pre-test. This made it necessary for us to intervene in the theoretical discussion, as well as to show that this kind of problem may be solved by using tree diagrams, as suggested by Totahasina (see Díaz and La Fuente, 2006).

The discussion was extended to a consideration on the usefulness and importance of showing pupils that by using two-way tables or/and tree diagrams, one can determine all the probabilities required in a descriptive way. In other words, the pupils can reach various conclusions just by relying on intuitive notions of probability so that the theory can be formalized later.

In their study, Silva, Kataoka and Cazorla (2008) demonstrated that the two-way table is a representational tool that allows one to carry out an easier conversion to a symbolic record of the probability of simple events, conditional probability and the probability of intersection. However, the teachers should be warned that for some probability problems the use of two-way tables is not adequate, because, as explained

by Tototashina (see Díaz and La Fuente, 2007), our understanding of the sequential nature of certain problems may be impaired by that kind of representation.

Further points discussed with the teachers were: i) the cognitive level of the students is influencing the depth of teaching concepts; ii) the link between theory and practice should be brought to the fore and could be taken up by questions like: How can information on an event that has already occurred be incorporated into the calculation of the probability of another event? Should events such as school and gender be regarded as dependent or independent?

Second Didactic Sequence

In the second didactic sequence, by manipulating dice, we resumed the discussion on conditional probability; we examined the concepts of dependent and independent events. Teachers worked in pairs to answer the following two questions. A die was actually handed out to them allow also for experimental or visual work.

1) Consider the following events in a throw of a die:

Event A – a number greater than 3 (face > 3) and Event B – an even number

Given that the number that appeared is greater than 3, what is the probability of it being an even number? In this situation, should events A and B be regarded as dependent or independent?

2) Consider the following events in a throw of a die:

Event A – a number less than 3 (face < 3) and Event B – an even number

Given that the number that appeared is less than 3, what is the probability of it being an even number? In this situation, should events A and B be regarded as dependent or independent?

Due to a discussion about conditional probability in the first didactic sequence, the answers to the first part of the question were satisfactory. Furthermore, in this inquiry, there was an argument on how the prior information, for instance a number less than 3, has a direct influence on the probability of an even number appearing, by the reduction of the sample space of the second event (Table 3).

Table 3. Different sampling spaces dependent on the result of the toss.

Condition or information on the toss	Sample space (S)	Probability of B : "even numbers"
None, prior to throwing	$\{1,2,3,4,5,6\}$	$3/6 = 1/2$
"face shown > 3 "	$\{4,5,6\}$	$2/3$
"face shown < 3 "	$\{1,2\}$	$1/2$

The debate about the independence of events – the second part of the question – was quite lengthy because most of the teachers (approx. 80%) had difficulties to understand the concept, which led to inadequate responses, since several teachers considered that the events were dependent in both cases, but in fact the events are dependent only in question 1) while in 2) they are independent. Some teachers stated that they regarded the events as dependent because in both questions for the probability of event A there were "already elements that were even numbers", and – remarkably enough – they did not even make any kind of calculation.

The confusion probably arises here because we are dealing with what Nabbout and Maury (2005) call a formal or stochastic independence of events (no reference is made to any experiments or chronology), which is based on its formal mathematical definition $P(A \cap B) = P(A).P(B)$.

Perhaps the theoretical confusion made by the teachers could be relieved if they were confronted, at first, with a situation called by that same author as chronologically or a-priori independent events (when two events are associated to experiences that occur successively, and their independence is postulated). “According to Maury (1985), the a priori independence does not present any need in the theory of probability, but most teachers and authors of textbooks refer to it, attempting to attach significance to the mathematical concept of independence” (see Nabbout, & Maury, 2005, p.1).

After the discussion of those types of independence of events and the way the calculations were set out, the teachers said to have understood the questions.

Third Didactic Sequence

A two-way table was presented with the following contextualization as an example of the initial display of the Bayes’ theorem:

Imagine that 300 women supposedly being pregnant underwent a test for diagnosing pregnancy with the following results (Table 4).

Table 4. Results of the pregnancy test for 300 women.

Pregnant	Diagnosis		Total
	Positive (B)	Negative (B')	
Yes (A)	216	24	240
No (A')	18	42	60
Total	234	66	300

Source: Carzola (2001).

The teachers were given the following question:

In our example, supposing that a randomly selected person received the test result and it is positive, what is the probability that the result is wrong?

The teachers were told that, in this situation, they should take into account that the group investigated consisted of women who thought they might be pregnant and that a sample of 300 had been chosen at random from this group.

Before attempting to find any kind of answer, the following concepts were discussed with the teachers:

- False negative – that is the probability that the test will not detect pregnancy when the woman in fact is pregnant (diagnostic error).
- False positive – that is the probability that the test will detect pregnancy when the woman in fact is not pregnant (diagnostic error).
- Specificity of a test – that is the probability that the result is negative given the fact that the woman is not pregnant.
- Sensitivity of a test – that is the probability that the result is positive given the fact that the woman is indeed pregnant.

After discussing these concepts, the teachers tried to work out these four probabilities by observing the results shown in Table 3. The first two probabilities regarding the false negative and the false positive were obtained quite quickly. However, the results regarding the specificity and sensitivity led to a lengthy discussion, even after they had supposedly understood the concepts in the first didactic sequence. For example, some of the teachers thought that the specificity ($P(B'|A')$) was equal to $42/300$, and the sensitivity ($P(B|A)$) equals to $216/300$ repeating the confusion between the concepts of conditional probability and the probability of intersection already presented in the pre-test and in the first didactic sequence.

With regard to the calculation of the probability of interest, that is, given the fact that a randomly selected person received the test result and it is positive, what is the probability that the result being wrong ($P(A'|B)$), our intervention was also needed. We demonstrated the result both by using the two-way table and the tree diagram (Figure 2). The diagram was also used to discuss the Total Probability Theorem in the calculation of $P(B)$, even though the teachers had found the result of 0.78 by observing directly Table 4.

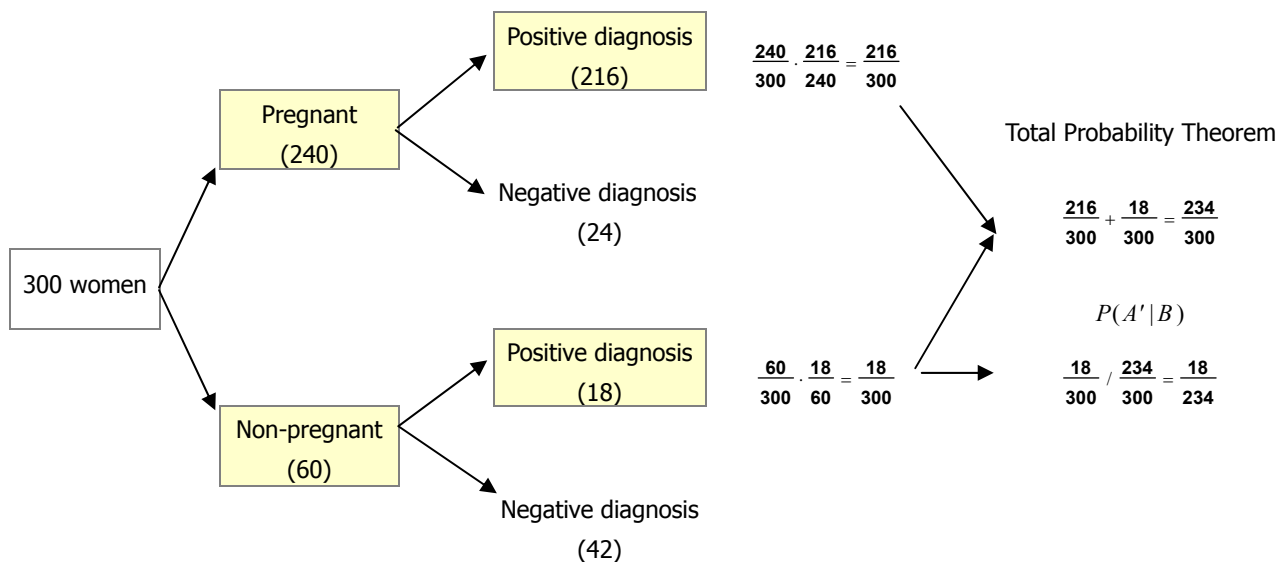


Figure 2. Tree diagram for the data shown in Table 3.

For the teachers, both the use of the tree diagram and the two-way table were enhancing for the resolution of the example. Then, we applied Bayes' theorem, both for this example involving only two events, and also in the general case with more events.

Although the teachers observed that the use of a formula was not required for this example, they stated that it was important to show it first with two events, and only then to progress to the generalization. Some of the teachers from Elementary II did not know Bayes' formula beforehand, but they thought that this kind of approach was useful and of potential value for working with students. This corroborates a statement by Figueiredo (2000), who suggested that both Bayes' Theorem and other contents of probability can be developed in an intuitive way – even in elementary schools – by means of tree diagrams and two-way tables.

After this preliminary discussion, the third didactic sequence was presented with a similar problem, this time about breast cancer (Pena, 2006), and with the difference that the data were not displayed in the form of a two-way table, but in a written form as follows:

Imagine that you come home and find your aunt in tears (she is older than 40), desperate because she had had a routine mammography and the result was positive.

You probably know that researches show that at this age range, 1% of women suffer from asymptomatic breast tumor. It is known that the mammogram shows a positive result in 80% of women who indeed have breast cancer, but a positive result is also found in 9.6% of the women who do not have cancer.

Based on this information, what is the probability that your aunt indeed suffers from breast cancer?

Some teachers commented that in this situation, unlike the example shown in Table 3, many women undergo preventive breast cancer examinations regardless of symptoms, and in the case of women over 40, this becomes, at least in theory, a mandatory annual examination, thus the ideal studied population would be in that age range. This meant that they regarded the “aunt” as someone well within that population in question.

In the course of that didactic sequence, after about 10 minutes we verified that most teachers had not managed to give a satisfactory answer; they reported difficulties to organize the data in a two-way table as well as to understand the factors that would condition the event as – differing from sequences 1 and 2 and unlike the example given in Table 3 – the question was not asked in a direct way. That is, in order to calculate all probabilities involved in this situation, the teachers had to perform a conversion from natural language in which the problem was presented, to either a reading table (graphic representation) or to symbols; such difficulties have also been observed by Silva et al (2008) as key problems in addressing this kind of questions. Therefore, our intervention was required for the initial development of the sequence, but it allowed the teachers to have an opportunity to look at the concepts again and discuss the results together.

Fourth Didactic Sequence

In order to start the discussion on the concepts of geometrical and frequentist probability, the teachers were asked the following questions:

A blindfolded person tries to hit a circular target with a radius of 40 cm and a bull’s eye at the center with a radius of 10 cm. If the person manages to hit the target with a throw, what is the probability that the person hits the bull’s eye?

What is the probability that a car will be stolen?

We asked the teachers to identify similarities or discrepancies between the two questions, and, after some discussion, they realized that the first question could be solved with the aid of geometrical concepts, while to solve the second question, it was necessary to observe how often the incident occurred.

The teachers also noticed that it was necessary to define other – complementary – information: For instance, in the first question, the distance of the person from the target and the height of the target – that is, to determine how randomness of the throw can be ensured, since just having the person blindfolded does not in itself guarantee the experiment to be random. For the second question, it was necessary to define the site of the investigation as well as how much time it would take to perform it.

After this stage, the concept of geometrical probability was discussed, a very old concept, used by Buffon in 1776 and again by Bertrand in 1889, and that, according to Tunala (1995), may be characterized by the fact that some probability problems correspond to a random selection of points in a sample space represented by geometrical figures. In the models in question, the probability of a specific event can be reduced to the relation between homogeneous geometrical measurements, such as length, area or volume.

Resuming the second question, it was pointed out to the teachers that through a large number of observations, one could obtain a probability of this event by dividing the number of times that the incident occurred by the number of observations carried out. The main feature of this approach is that the mathematical value of the probability emerges from the experimental process, thus characterizing the so-called frequentist probability.

We ended the theoretical discussion by stressing that some probability problems may be solved by adopting a geometrical approach, where the result can be interpreted and checked in the original approach – that of probability. The idea was to introduce geometrical probability, as well as a confrontation of results with the concept of frequentist probability. In that way, the fourth and fifth didactic sequences were carried out in practice.

In the fourth sequence, we performed Buffon's needle experiment, in which a needle of length l is thrown onto a surface with parallel lines separated by a distance d (with $d \geq l$). The probability of the needle crossing a line is asked for (see also Tunala, 1995); we employed geometrical and probabilistic concepts to estimate this probability. The charm of the context lies in the fact that the resulting probability is a function of π :

$$P(\text{needle crosses a line}) = \frac{2 \cdot l}{d \cdot \pi}$$

If you approximate this probability by the relative frequencies of a series of trials, then you may get an estimate of the value of π – a paradigm for non-random entities – by means of this classic random experiment – somehow a paradox, which is always also confirmed by participants and it was also puzzling our teachers. To start with the sequence, we briefly told the story of the French mathematician and naturalist George Louis Leclerc, the Count of Buffon, and the origin of the problem. We also showed the results of simulations carried out by other researchers such as Wolf (1850), Smith (1855), Lazzerini (1901), or Reina (1925). The discussion on those results was really interesting because the teachers were able to observe the values obtained as an approximation of π for different numbers of throws and combination of needle sizes, and the distance between parallel lines.

Next, pairs of teachers carried out the activity step by step, making use of the room floor to determine the parallel straight lines (Figure 3 left). Each pair performed 50 throws, with results ranging from 2.709 to 4.200. The teachers were enthusiastic about the activity, and it was interesting to see their expectations over the results, both from each pair and collectively ($\hat{\pi} = 3.1543$) for the approximation of π (Figure 3 right).



Figure 3. Teachers from Elementary II School performing the fourth didactic sequence, Buffon's needle experiment.

We also showed two simulations that were carried out with the software R (R Development Core Team, 2007), using the animation package and the buffon.needle feature. A simulation with 50 throws and another with 10,000 throws resulting in completely different estimations of π (Figure 4). In addition, we carried out an online simulation available at <http://www.mste.uiuc.edu/reese/buffon/buffjava.html>. It was interesting to start a discussion on the phenomenon of convergence of estimations, based on the law of large numbers.

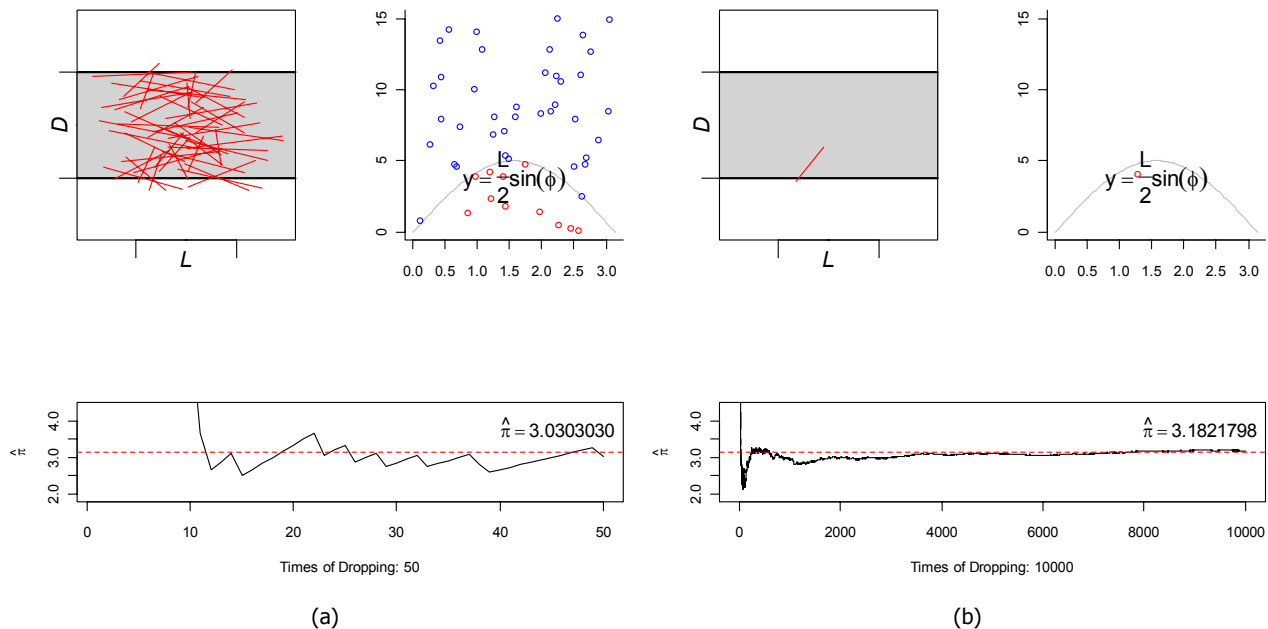


Figure 4. Simulations of 50 (a) and 10,000 (b) throws of a needle with $l = 10$ and $d = 30$ in the software R - animation package_

Fifth Didactic Sequence

The fifth didactic sequence dealt with the “spaghetti” problem (Wagner, 1997):

- A piece of spaghetti was given to each of the participants (n) and they were asked to break it into 3 pieces at random, without explaining the purpose behind to them;
- Next, they were asked to try to form a triangle from the 3 pieces;
- We noted the number of successes (for triangles) obtained (k) and calculated the value of $P_F = \frac{k}{n}$

We compared the value obtained by the frequentist probability – in our experiment we got $P_F = 0.67$ – to the geometrical probability – in this case equal to 0.25. Due to time constraints, it was not possible to outline the way the value for the geometrical probability was obtained and we had to refer the teachers to the material used by Wagner (1997). They understood the time constraint, but were curious about the geometrical explanation. At first, the teachers were very concerned about the difference that had been obtained, and thought that a problem had happened while carrying out the experiment, that is, they were in doubt about whether the geometrical and/or frequentist procedures were correct.

As the teachers were unable to explain what had happened, it was necessary for us to intervene. We explained that the occurrence was due to factors inherent to the experiment, since it is normal for people to break a piece of spaghetti into three parts that are all about the same size (they break the spaghetti first into two pieces and tend to take the longer stick for the second break), and one would rarely obtain three pieces that do not form a triangle (Figure 5).

We also discussed whether in those cases the use of the frequency procedure would lead to a false result, even if a large number of observations had been investigated, because the basic principle in conducting an experiment is to ensure that it is really undertaken in a random way to avoid obtaining biased responses.

To conclude, in carrying out those two last didactic sequences, we emphasized the importance of randomness by drawing a parallel between the results of geometrical and frequentist probability. A further advantage of the activity was seen in its potential to explore mutually related concepts such as estimates, the variability of small samples, and the use of games in simulation processes.

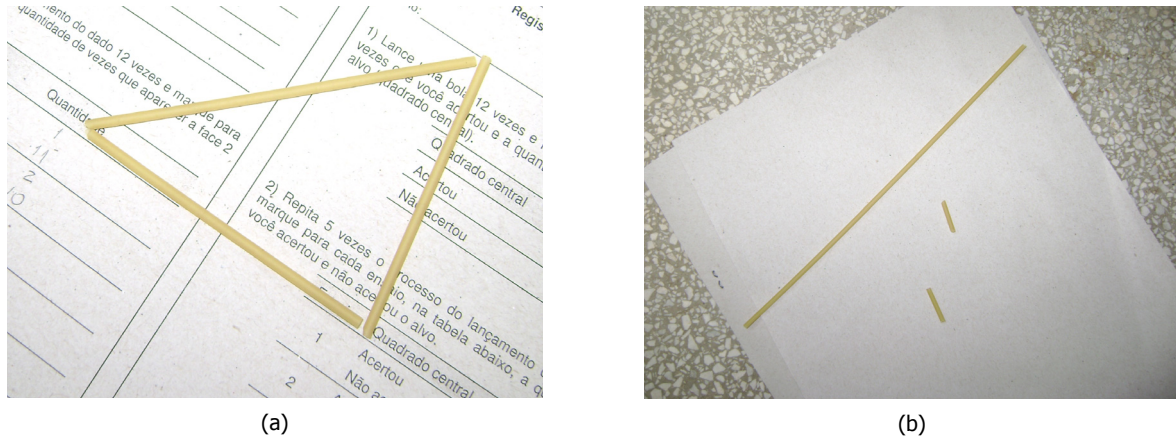


Figure 5. Examples of (a) success, and (b) failure in the spaghetti experiment.

Evaluation of the Workshop

Final Assessment of the 3rd Session

After the five didactic sequences, we asked the teachers to work on the pre-test (shown in Table 1) again and found that, in most cases, their approach for tackling problems and reasoning had changed a great deal. Only two teachers were still uncertain about which had been the conditional event, that is, they still did not know whether they should investigate the probability of accidents to be fatal due to the fact that the driver was drunk, or to calculate the probability that the driver was drunk in cases when there were fatal victims. We then closed the meeting with a group discussion about how to solve this question.

It was also evident from the analytical reports carried out by the teachers themselves that all the discussions were valuable and that all the activities could feasibly be incorporated into classroom practice. This was partly because some of the teachers spoke about their initial difficulties in understanding some of the concepts, such as conditional probability and the independence of events and how to connect them to other subjects.

Final Assessment of the Pedagogical Workshop

As had already been found from the analysis of the entrance questionnaire, most of the teachers stated that, although they had a sound knowledge of mathematics acquired during their basic education and professional training, they were unfamiliar with the basic concepts of probability and statistics. This was due to the slight contact with those areas during their teacher training, as well as the fact that their teaching experience had given them little opportunity to get familiarized with didactic features connected to teaching probability and statistics.

In general terms, the qualitative assessment of the results obtained from the workshop confirm that the teachers acquired substantial knowledge on the worked contents, as well as it evidenced that their pedagogical practice had been modified, from their testimonials. The teachers completed an evaluation form on the workshop (Table 5).

Table 5. Evaluations from pedagogical workshops delivered to 30 mathematics teachers, Elementary II and Secondary School.

Questions	Excellent (%)	Good (%)	Regular (%)
Practical application of the content, motivation	70.00	30.00	-
Clarity shown in reviewing the content	73.34	26.67	-
Command of the content on the part of lecturers	86.67	13.34	-
Concern about group learning	80.00	20.00	-
Communication and interaction with the group	83.33	16.67	-
Tell your learning level with the class	40.00	53.33	6.67
The choice of content was appropriate	66.67	33.33	-

The teachers contributed a number of suggestions in the teachers' evaluation form such as, for instance, that further courses should be delivered on subjects related to probability and statistics, always prioritizing the interdisciplinary character, reinforcing the need for updating and qualification of mathematics teachers. One teacher made the following statement:

"The course was very useful in drawing our attention to daily information received through magazines and other media, and for making us more participating citizens, with a critical sense. And the most important thing was qualifying us to use the results in teaching."

It is worth mentioning that, after the workshop, we met many of the teachers who told us that they had used some of the sequences with their students with very positive and promising results.

Final Considerations

We deem the results attained during the workshops as rather positive, but we found the need to prepare further workshops focused both on probability and on statistics, as well as planning other didactic sequences which would allow teachers to upgrade their probabilistic reasoning.

We also verified that it would be important to have a post-workshop stage, by following-up those teachers at school to verify the students' response to such activities and, therefore, allowing for an effective improvement in the teaching-learning process, which, for its own characteristics, should be cyclical. That is, the exchange of experiences with teachers should not finish at the end of the workshop itself in order to allow further cycles of improvement of the activities and to get the teachers continuously involved into issues of their professional improvement.

Lastly, this work may be considered as a pilot-plan, with real possibilities for further investigation. We also expect that the work will serve, at least, as an indication for actions by other researchers in different places. Therefore, we emphasize that researches and projects really matter for the continued formation of mathematics teachers in probability.

Moreover, our experience with the workshop shows that there are still great intellectual and

governmental investments to be done so that some changes may diffuse from the didactical books into the process of formation of mathematics teachers and their actual teaching, in a way that the PCN recommendations about teaching statistics and probability in fact can be implemented sustainably and faster in the Brazilian school system.

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Annex A – Pre-Test

Federal University of Lavras

UFLA, Dept of Exact Sciences

Post-graduate program in Statistics and Agriculture Research

School: ☐ Public ☐ Private

1) The Internet site for the Brazilian Diabetes Society (SDB) asked the following question to specialists and persons who sufferer from diabetes: Are you in favor of or are you against the use of frozen embryos to obtain stem cells that can be used in research?

The results from a sample of 955 people were

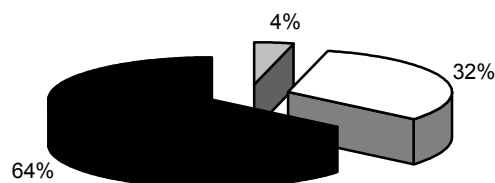
614 (64%) in favor of freedom of research, while

303 (32%) were against and

34 (4%) did not know what their opinion was.

Research findings from the Brazilian Diabetes Society published 4/11/2004.

<http://www.diabetes.org.br/imprensa/noticiasnacionais/index.php?mat=200400620#top>



Do you think that the results of the survey can be regarded as the opinion of all Brazilians?
Explain the reasons for your reply.

2) Research from the exit poll carried out by IBOPE (Brazilian Institute of Public Opinion and Statistics) on 1st October (Tables 1A and 2A).

Table 1A. Forecast for the presidential elections.

Candidate	(%)
Lula -13- PT	50
Geraldo Alckmin - 45- PSDB	38
Heloísa Helena - 50 -PSOL	8
Cristovam Buarque - PDT	3
Ana Maria Rangel - PRP	1
Luciano Bivar - 17 - PSL	0
José Maria Eymael - 27- PSDC	0
Rui da Costa Pimenta - 29 - PCO	0

Sample: 60,300 people on the electoral roll were interviewed in 445 Brazilian municipalities.

Margin of error: two percentage points either way with a 99% degree of confidence.

http://www.ibope.com.br/calandraWeb/servlet/CalandraRedirect?temp=6&proj=PortalIBOPE&pub=T&db=cald&comp=pesquisa_leitura&docid=195A9D286600AF5E832571FA00771199

On the basis of the data, it can be stated that:

Lula can be elected in the first round
or He will have to run against Geraldo Alckmin in a second round.

Table 2A.Voting forecast for the position of Governor, Rio de Janeiro.

Candidate	(%)
Sergio Cabral - 15 - PMDB	41
Denise Frossard - 23 - PPS	23
Marcelo Crivella - 10 - PRB	21
Eduardo Paes - 45 - PSDB	5
Vladimir - 13 - PT	5
Lupi - 12 - PDT	2
Alexandre Furtado - 17 - PSL	1
Luiz Novaes - 27 - PSDC	1
MiltonTemer – 50 - PSOL	1
Eliane Cunha – 44 - PRP	0
Thelma Maria – 29 - PCO	0

Sample: 6,000 people on the electoral roll were interviewed in the municipalities.

Margin of error: two percentage points either way with a 99% degree of confidence.

On the basis of the data presented, can it be stated who will run against Cabral in the 2nd Round?
If so, who will be candidate?

3) Table 3A below shows statistics from the State Highway Dept., providing information about the number of accidents with victims (fatal or otherwise), and the condition of the main motorist involved (drunk or sober)

Table 3A. Statistics from the State Highway Dept. regarding the number of victims.

Driver/Victim	non-fatal	fatal
Sober	1228	275
Drunk	239	76

Would you say that the condition of the driver – drunk or sober – affects the occurrence of fatalities?