Students' Biases in Conditional Probability Reasoning

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Summary

In this paper, we present results from a study aimed to assess potential relationships between biases in conditional probability reasoning (fallacy of the transposed conditional, fallacy of the time axis, conjunction fallacy, base rate fallacy and confusing independence and mutually exclusiveness) and formal knowledge of the concepts. Factor analysis of the responses by 414 psychology students to a questionnaire suggest that the psychological biases investigated are unrelated to conditional probability problem solving performance in these students. Comparison of responses of two groups before and after instruction shows systematic improvement in problem solving capacity but little change in part of the items related to psychological biases.

Introduction

Conditional probability and Bayesian reasoning are important components in undergraduate statistics, since they intervene in the understanding of classical and Bayesian inference, linear regression and correlation models, multivariate analysis, and other statistical procedures, which are frequently used in professional work and empirical research. Conditional probability reasoning is also a crucial part of statistical literacy, since it helps making accurate decisions or inferences in everyday life. However, several studies show psychological biases related to this concept. Most of these biases have been studied in isolation, so that it is unclear whether these incorrect types of reasoning are related to each other or to formal mathematical knowledge. In our investigation, we intend to explore potential relationships between the biases described in the literature and whether the biases decrease with instruction.

Biases related to conditional probability reasoning

Research on the understanding of conditional probability has been carried out with both secondary school and university students. Fischbein and Gazit (1984) conducted teaching experiments with children in grades 5-7 (10-12 year-olds) and compared two types of problems involving experiments with and without replacement. They found that conditional probability problems were harder for these children in "without replacement" as compared to "with replacement" situations. Following that research, Tarr and Jones (1997) identified four levels of thinking about conditional probability and statistical independence in middle school students (9-13 year-olds):

- Level 1 (subjective): Students ignore given numerical information in making predictions; they use subjective reasoning in assessing conditional probability and independence.
- Level 2 (transitional): Students demonstrate some recognition of whether consecutive events are related

or not; however, their use of numbers to determine conditional probability is inappropriate.

- Level 3 (informal quantitative): Students' differentiation of "with and without replacement situations" is imprecise as is the quantification of the corresponding probabilities; they are also unable to produce the complete composition of the sample space in judging independence.
- Level 4 (numerical): Students state the necessary conditions for two events to be related, they assign the correct numerical probabilities and they distinguish between dependent and independent events in "with replacement" and "without replacement" situations.

Even when students progress towards the upper level in this classification, difficulties remain at high school and university (Tarr & Lannin, 2005). Below, we summarize various studies:

Confusing causal and diagnostic situations: From a psychological point of view, the person who assesses a conditional probability *P(A/B)* may perceive different types of relationship between *A* and *B* depending on the context (Tversky and Kahneman, 1982a). If *B* is perceived as a cause of *A, P(A/B)* is viewed as a causal relation, and if *A* is perceived as a possible cause of *B, P(A/B)* is viewed as a diagnostic relation. People tend to overestimate causally perceived conditional probabilities while they ignore diagnostic conditional probabilities. Moreover, some people confuse diagnostic and causal probabilities; this is a particular case of confusing *P(A/B)* and *P(B/A)* that was termed as the *fallacy of the transposed conditional* (Falk, 1986).

The fallacy of the time axis: Falk (1989) found that students are reluctant to believe that an event could condition another event that occurs before it. Similar results are reported by Gras and Totohasina (1995) who introduce the term *chronological conception* for the case of students who interpret the conditional probability P(*A/B*) as a temporal relationship and think that the conditioning event *B* should always precede event *A*.

The base rate fallacy: Early research by Tversky and Kahneman (1982a) on reasoning needed to solve problems involving Bayes' theorem, suggests that people do not employ this reasoning intuitively when approaching these problems. Their research reveals the robustness and wide extent of the base-rate fallacy in students and professionals, that is, the failure to take base rates into account when judging probabilities. In Bayes' problems you are given statistics for a population as well as for a particular part of the population; both types of information has to be considered together to solve the problem; however, people tend to ignore the population base rate (Bar-Hillel, 1983; Koehler, 1996).

Synchronic and diachronic situations: In diachronic situations the problem is formulated as a series of sequential experiments, which are carried out over time. Synchronic situations are static and do not incorporate an underlying sequence of experiments. Formally, the two situations are equivalent, however, empirical research shows that individuals may perceive them differently. Sánchez and Hernández (2003) find that 17 to 18 year-old students do not always perceive the situations as equivalent (n=196). These students add probabilities instead of using the product rule in computing a compound probability in a synchronic format but use the correct rule in a diachronic format of the problem.

Tversky and Kahneman (1982a) coined the term *conjunction fallacy* for people's unawareness that a compound probability cannot be higher than the probability of each single event. Finally, some students confuse *independence* with *mutually exclusiveness* (Sánchez, 1996; Truran & Truran, 1997).

Frequency versus probabilistic format: Recent research suggests that conditional probability problems are simpler when information is given in natural frequencies, instead of using probabilities, percentages or relative frequencies (Gigerenzer, 1994; Gigerenzer & Hoffrage, 1995; Sedlmeier, 1999; Martignon, &

Wassner, 2002). However, our own research in a small experimental setting shows that the majority of the students is able to solve problems proposed in probability format, when they are taught to use a Bayes' table and Excel to facilitate the computations (Díaz & Batanero, 2007; *n*=75 psychology students). A short questionnaire also corroborates an intuitive understanding of the theorem and its utility in updating conditional probabilities. From these results we conclude that is it possible to teach students more productive and generalizable methods of solving conditional probability problems with data given in probabilistic format. We also think this approach is needed in the teaching of many professionals, including psychology students who should deal with conditional probability in situations such as evaluation or diagnosis.

Method

In spite of the amount of previous studies related to conditional probability reasoning, we found no comprehensive questionnaire to globally assess students' understanding and misconceptions on these topics. Following this need, we built a comprehensive instrument that may be used to assess the different biases and misunderstanding related to conditional probability. The construction of the CPR questionnaire (Conditional Probability Reasoning) is described in detail in Díaz and de la Fuente (2006, 2007).

The questionnaire (a translation is included in the Appendix) was administered to different samples of students majoring in Psychology who took an introductory statistics course in the first year of University studies (18-19 year-olds) at the Universities of Granada, Murcia, Jaén and Huelva. All of them had studied conditional probability at secondary school level. To explore our conjecture that biases on conditional probability reasoning are unrelated to mathematical performance in the tasks, we carried out a factor analysis of the set of responses to all the items in the CPR questionnaire by 404 students (Granada, n=208 students and Murcia n=196 students). These students had previously been taught conditional probability and Bayes' theorem for about 2 weeks, including the use of tree diagrams and two-way tables to solve problems as part of their statistics course, before they completed the questionnaire. We used the most conservative factor extraction method (principal components) and an orthogonal rotation (varimax) in order to avoid distorting the data structure.

To check our second conjecture that biases do not improve much with instruction, we used discriminant analysis to compare the performance of the 208 students from Granada (who had previous instruction in conditional probability) with those of another group (177 students psychology from the universities of Huelva and Jaen) who took the questionnaire before their course in conditional probability. Even if the students in our sample are majoring in psychology, their social and educational characteristics are not different from other students in an introductory statistics course. We therefore consider our findings could be of interest to statistics lecturers in a wider range of specialties.

Results

Structure of responses

In Table 1 we present the factor loadings (correlations) of items with the different factors after rotation (only values higher than .3 are shown). We identified seven factors with eigenvalues higher than 1 that explained 59% of the total variance: 21% the first factor, 7 % the second factor, and about 6% in each of the remaining factors. This result confirms the specificity of each item, and the multidimensional character of understanding conditional probability.

It also reveals the higher relevance of the first factor to which most of the open-ended problems contribute, in particular Bayes' problems, total probability and compound probability problems. Solving any of these problems requires at least two stages, in the first of which a conditional probability is computed, which is used in subsequent steps (e.g. product rule). We interpret this factor as *ability to solve complex conditional probability problems.* Computing simple, joint and conditional probability from a two-way table (item 1) appears as a separate factor, probably because the *task format* affected performance, a fact which has also been noticed by Ojeda (1996), Gigerenzer (1994) and Lonjedo and Huerta (2005), amongst other researchers. A third factor reflects the relationships between definition, sample space and computation of conditional probabilities in "with and without replacement" situations, which suggests that it may be identified with *Level 4 reasoning* in the classification by Tarr and Jones (1997).

The remaining factors point out to the different biases affecting conditional probability reasoning, which we describe in the introduction, and which appear unrelated to mathematical performance in problem solving understanding (Factor 1), computing probabilities from a two-way table (Factor 2), and to Tarr and Jones's (1997) level 4 reasoning (Factor 3) as the related items are not included in the first three factors. Each of the biases (transposed conditional, time axis fallacy, conjunction fallacy, independence/mutually exclusiveness) also appear unrelated to one another; in some cases some of them are opposed or related to some mathematical components of understanding conditional probability. For example, independence is linked to the base rate fallacy problem (where people have to judge whether the events are independent or not) and opposed to the idea of dependence.

Table 1. Factor Loadings for Rotated Components in Factor Analysis of Responses to Items

Item	Factor						
	1	2	3	4	5	6	7
18. Solving Bayes' problem	.77						
14. Solving total probability problem	.76						
17. Solving product rule problem, in case of dependence	.76						
16. Solving product rule problem, in case of independence	.67						
15. Solving a conditional probability problem, in case of independence	.43		.42				
12. Describing the restricted sample space	.40		.46				
2. Base rate fallacy	.34						.48
1b. Computing conditional probability from a 2- way table	.32	.61					
1c. Computing joint probability, from a 2- way table		.79					
1a. Simple probability, from a 2- way table		.61					
1d. Computing inverse conditional probability from a 2- way table		.77					
13. Solving a conditional probability problem, in a single experiment			.67				
11. Defining conditional probability and giving an example			.59				
4. Solving a conditional probability problem, in case of dependence			.39	.44			
9b. Time axis fallacy				.71			
8. Time axis fallacy				.70			
10. Solving a joint probability problem in diachronic experiments				.35		-.46	
5. Computing conditional probability from joint and compound probability					.66		
6. Conjunction fallacy					.62		
7. Transposed conditional /causal-diagnostic						-.65	
9a. Computing conditional probability, dependence						.66	
3. Independence /mutually exclusiveness							.68

Improvement with instruction

Discriminant analysis is used to study improvement after instruction, using the groups (instruction or no instruction) as criterion for discrimination. Several statistics (Wilks' Lambda= 0.63; Chi-Square χ^2=171.117; canonical correlation=0.697; 82.3% students correctly classified) reveal a high discriminant power of the CPR questionnaire.

In Table 2 we present percentages of correct responses to each item in both groups and p-values for the differences between the two groups. Differences always favour the group with instruction and are statistically significant, with exception of the items assessing the conjunction fallacy, the fallacy of transposed conditional, understanding of the difference between causal and diagnostic reasoning, and one of the items measuring the fallacy of time axis. Note that, even if the difference is statistically significant in the other item assessing the time axis fallacy, in fact results for that item are worse after instruction. Consequently, the CPR questionnaire may serve to discriminate between students with and without specific instruction in conditional probability if we exclude part of the items assessing the psychological biases.

Note that there was an important improvement in performance in all open-ended problem solving tasks: items 13 (Solving a conditional probability problem, in a single experiment), item 14 (solving a total probability problem), item 15 (solving a conditional probability problem, in case of independence), item 16 (solving a product rule problem, in case of independence), item 17 (solving a product rule problem, in case of dependence), and item 18(solving a Bayer's problem). Other items with good discrimination were item 1 (computation of probabilities from a 2-way table), item 11 (defining conditional probability and giving examples of it) as well as two items assessing psychological biases (item 2, base rate fallacy and item 3, distinguishing between independence and mutually exclusiveness).

Table 2. Percentages of correct responses to items in students with and without instruction

Item	No instruction (n=177)	Instruction (n=208)	p-value
1b. Computing conditional probability from a 2- way table	67	94	0.000
1c. Computing joint probability, from a 2- way table	29	63	0.000
1a. Simple probability, from a 2- way table	35	69	0.000
1d. Computing inverse conditional probability from a 2- way table	37	70	0.000
2. Base rate fallacy	33	53	0.000
3. Independence /mutually exclusiveness	23	41	0.000
4. Solving a conditional probability problem, in case of dependence	77	89	0.001
5. Computing conditional probability from joint and compound probability	37	48	0.042
6. Conjunction fallacy	21	24	0.465
7. Transposed conditional /causal-diagnostic	35	35	0.989
8. Time axis fallacy	8	13	0.142
9a. Computing conditional probability, dependence	72	81	0.050
9b. Time axis fallacy	37	25	0.009
10. Solving a joint probability problem in diachronic experiments	62	76	0.002
11. Defining conditional probability and giving an example	17	47	0.000
12. Describing the restricted sample space	56	67	0.050
13. Solving a conditional probability problem, in a single experiment	20	33	0.005
14. Solving total probability problem	18	55	0.000
15. Solving a conditional probability problem, in case of independence	34	75	0.000
16. Solving product rule problem, in case of independence	25	56	0.000
17. Solving product rule problem, in case of dependence	24	59	0.000
18. Solving Bayes' problem	20	49	0.000

Conclusion and further research

Our results indicate that students' performance in problem solving ability and formal understanding of conditional probability improved with instruction; we observed a high percentage of correct solutions to all the problems (including total probability and Bayes' problems). However, some of the biases described in the literature are widespread in these students' thinking and do not improve with instruction.

The results of the factor analysis reflect the complex relationship between probabilistic concepts and intuition (Borovcnik, Bentz & Kapadia, 1991; Borovcnik & Peard, 1996): items assessing biases in conditional probability reasoning are unrelated to those assessing formal knowledge and are partly unrelated to each other. These results are consistent with Batanero, Henry and Parzysz (2005) who trace an analogous lack of intuition in the historical development of the discipline. Even though independence and conditional probability are informally used from the very beginning of the study of chance games, it is not before the middle of the 18th century that these two concepts are made explicit in the mathematical theory. Furthermore, von Mises (1928, 1952) criticizes the formal modern definition of independence because as it is not intuitive at all. It is natural that these historical difficulties recur in the students' learning of probability.

Consequently, our research indicates the need for reinforcing the study of conditional probability in teaching data analysis at university level. Moreover, it yields strong reasons for a change of teaching approaches. As suggested by Feller (1973, p. 114) "the notion of conditional probability is a basic tool of probability theory, and it is unfortunate that its great simplicity is somewhat obscured by a singularly clumsy terminology". We also suggest to follow Nisbett and Ross' recommendations (1980), according to which students should be "given greater motivation to attend closely to the nature of the inferential tasks that they perform and the quality of their performance" (p. 280) and consequently "statistics should be taught in conjunction with material on intuitive strategies and inferential errors" (p. 281). The items included in the CPR questionnaire may serve instructors either to assess the extent of these biases among their students or to build up didactic situations where students are confronted with their misconceptions about conditional probability.

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Appendix: English translation of the CPR questionnaire

Item 1. (Estepa, 1994). In a medical centre a group of people were interviewed with the following results:

	55 years-old or younger	Older than 55	Total
Previous heart stroke	29	75	104
No previous heart stroke	401	275	676
Total	430	350	780

Suppose we select at random a person from this group:
   a. What is the probability that the person had a heart stroke?
   b. What is the probability that the person had a heart stroke and, at the same time is older than 55?
   c. When the person is older than 55, what is the probability that he/she had a heart stroke?
   d. When the person had a heart stroke, what is the probability of being older than 55?

Item 2. (Tversky & Kahneman, 1982a). A witness sees a crime involving a taxi in a city. The witness says that the taxi is blue. It is known from previous research that witnesses are correct 80% of the time when making such statements. The police also know that 15% of the taxis in the city are blue, the other 85% being green. What is the probability that a blue taxi was involved in the crime?

   a. $\dfrac{80}{100}$

   b. $\dfrac{15}{100}$

   c. $\dfrac{15}{100} \times \dfrac{80}{100}$

   d. $\dfrac{15 \times 80}{85 \times 20 \ + \ 15 \times 80}$

Item 3. (Sánchez, 1996). A standard deck of playing cards has 52 cards. There are four suits (clubs, diamonds, hearts, and spades), each of which has thirteen cards (2,..., 9, 10, Jack, Queen, King, Ace). We pick a card up at random. Let A be the event "getting diamonds" and B the event "getting a Queen". Are events A and B independent?
   a. A and B are not independent, since there is the Queen of diamonds.
   b. A and B are only then independent when we first get a card to see if it is a diamond, return the card to the pack and then get a second card to see if it is a Queen.
   c. A and B are independent, since P(Queen of diamonds)= P(Queen) × P(diamonds).
   d. They A and B are not independent, since P(Queen | diamonds) $\neq$ P(Queen).

Item 4. There are four lamps in a box, two of which are defective. We pick up two lamps at random from the box, one after the other, without replacement. Given that the first lamp is defective, which answer is true?
   a. The second lamp is more likely to be defective.
   b. The second lamp is most likely to be correct.
   c. The probabilities for the second lamp being either correct or defective are the same.

Item 5. (Eddy 1982). 10.3 % of women in a given city have a positive mammogram. The probability that a woman in this city has both positive mammogram and a breast cancer is 0.8%. A mammogram given to a woman taken at random in this population was positive. What is the probability that she actually has breast cancer?

    a. $\dfrac{0.8}{10.3} = 0.0776$, 7.76 %
             b. $10.3 \times 0.8 = 8.24$, 8.24 %
             c.   0.8 %

Item 6. (Tversky & Kahneman, 1982 b). Suppose a tennis player reaches the Roland Garros final in 2005. He has to win 3 out of 5 sets to win the final. Which of the following events is more likely or are they all equally likely?

    a.   The player will win the first set.
    b.   The player will win the first set but lose the match.
    c.   Both events a. and b. are equally likely.

Item 7. (Pollatsek et al. 1987). A cancer test is administered to all the residents in a large city. A positive result is indicative of cancer and a negative result of no cancer. Which of the following results is more likely or are they all equally likely?

    a.   A person has cancer if they get a positive result.
    b.   To have a positive test if the person has cancer.
    c.   The two events are equally likely.

Item 8. (Ojeda 1996).We throw a ball into the entrance E of a machine (see the figure). If the ball leaves the system through exit R, what is the probability that it passed by channel I?

    a. $\frac{1}{2}$
                 b. $\frac{1}{3}$

    c. $\frac{2}{3}$
                 d. Cannot be computed
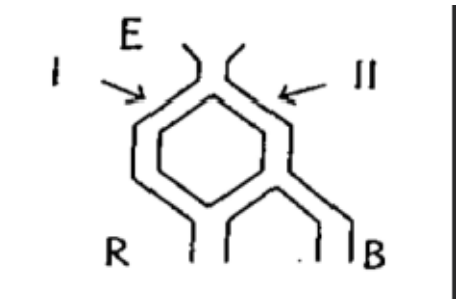
Item 9. (Falk, 1986, 1989). Two black and two white marbles are put in an urn. We pick a white marble from the urn. Then, without putting the white marble into the urn again, we pick a second marble at random from the urn.

If the first marble is white, what is the probability that this second marble is white? $P(W_2 \mid W_1)$

    a. $\frac{1}{2}$
           b. $\frac{1}{6}$
           c. $\frac{1}{3}$
           d. $\frac{1}{4}$

If the second marble is white, what is the probability that the first marble is white? $P(W_1 \mid W_2)$

    a. $\frac{1}{3}$
           b. Cannot be computed
           c. $\frac{1}{6}$
           d. $\frac{1}{2}$

Item 10. An urn contains one blue and two red marbles. We pick up two marbles at random, one after the other without replacement. Which of the events below is more likely or are they equally likely?
  a. Getting two red marbles.
  b. The first marble is red and the second is blue
  c. The two events a) and b) are equally likely.

Item 11. Explain in your own words what a simple and a conditional probability is and provide an example.

Item 12. Complete the sample space in the following random experiments:
  a) Observing gender (male/female) of the children in a three children family (e.g. *M F M* ,...)
  b) Observing gender of the children in a three children family if two or more children are male.

Item 13. In throwing two dice the product of the two numbers is 12.
What is the probability that none of the two numbers is a six (we take the order of the numbers into account).

Item 14. 60% of the population in a city are men and 40% women. 50% of the men and 35% of the women smoke. We select a person from the city at random; what is the probability that this person is a smoker?

Item 15. A person throws a die and writes down the result (odd or even). It is a fair die (that is, all the numbers are equally likely). These are the results after 15 throws:
  Odd, even, even, odd, odd, even, odd, odd, odd, odd, even, even, odd, odd, odd.
The person throws once more. What is the probability to get an odd number this time?

Item 16. A group of students in a school take a test in mathematics and one in English. 80% of the students pass the mathematics test and 70% of the students pass the English test. Assuming the two subjects score independently, what is the probability that a student passes both tests (mathematics and English)?

Item 17. According to a recent survey, 91% of the population in a city usually lie and 36% of those usually lie about important matters. If we pick a person at random from this city, what is the probability that the person usually lies about important matters?

Item 18. (Totohasina 1982). Two machines M_1 and M_2 produce balls. Machine M_1 produces 40 % and M_2 60% of balls. 5% of the balls produced by M_1 and 1% of those produced by M_2 are defective. We take a ball at random and it is defective. What is the probability that that ball was produced by machine M_1?