21st LL-Seminar on Graph Theory

Universität Klagenfurt
und
Montanuniversität Leoben

April 26–28, 2004

Foreword

More than twenty years ago graph theorists from Leoben and Ljubljana began to meet informally but regularly in the "Ljubljana-Leoben Seminar on Combinatorics". The iron curtain was not so tight any more and the short distance between Ljubljana and Leoben made it easy to cooperate in those dark pre-internet times. Within the years a fruitful collaboration arose and the Seminar kept growing. Since 1989 we even have an official program for these meetings.

Although organized mainly from Leoben and Ljubljana the seminar took place in many locations in Slovenia and Austria and we are grateful that the University of Klagenfurt, halfway between Leoben and Ljubljana, hosts it this time.

We thank all participants for their interest and their contributions to the scientific program, and are looking forward to an interesting, productive and enjoyable meeting!

Franz Rendl, Klagenfurt

Wilfried Imrich, Leoben

Bojan Mohar and Tomaž Pisanski, Ljubljana

Sandi Klavžar and Boštjan Brešar, Maribor

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Program

Monday, April 26, 2004

11:15	11:30	Opening and welcome addresses	

11:30	12:00	Norbert Seifter	*Transitive Digraphs*

12:00	14:00	Lunch Break	

14:00	14:30	Sandi Klavžar	Θ-*graceful labelings of partial cubes*
14:30	15:00	Josef Leydold	*Faber-Krahn Type Inequalities for (Non-regular) Trees*

15:00	15:30	Coffee Break	

15:30	16:30	Peter Mihók	*Additive and hereditary properties of systems of objects*

16:30	18:00	Workshops	

19:00	Conference Dinner Restaurant Weidenhof am See

Tuesday, April 27, 2004

09:00	09:30	Igor Dukanović	*A Semidefinite Programming Based Heuristic for Graph Coloring*
09:30	10:00	Janez Povh	*How to approximate the bandwidth of a graph using semidefinite programming ?*

10:00	10:30	Coffee Break

10:30	11:00	Sergio Cabello	*Planar embeddability of the vertices of a graph using a fixed point set is NP-hard*
11:00	11:30	Iztok Peterin	*On almost-median graphs*
11:30	12:00	Wilfried Imrich	*On edge-preserving maps of graphs*

12:00	14:00	Lunch break

14:00	15:00	Problem session

15:30	Excursion - Herzogstuhl - Burg Hochosterwitz - Magdalensberg

Wednesday, April 28, 2004

09:00	09:30	Boštjan Brešar	*On Integer Domination in Graphs and Vizing-like Problems*
09:30	10:00	Janez Žerovnik	*Weak Reconstruction of Strong Product Graphs*

10:00	10:30	Coffee Break

10:30	11:00	Drago Bokal	*Circular chromatic number of oriented hexagonal systems*
11:00	11:30	Aleksander Vesel	*Characterisation of the Resonance Graphs of Catacondensed Hexagonal Graphs*
11:30	12:00	Petra Žigert	*Fibonacci cubes are the resonance graphs of fibonaccenes*

12:00	End of lectures and lunch

Abstracts

Abstracts are listed alphabetically with respect to PRESENTING AUTHOR.

Circular chromatic number of oriented hexagonal systems

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The circular chromatic number of arbitrary orientation of any linear hexagonal system is determined. It depends on the existence of certain oriented substructure and is always of the form $q = \frac{5k+1}{4k+1}$ where $1 \leq k \leq \frac{n}{2}$ is an integer and n is the number of hexagons.

On Integer Domination in Graphs and Vizing-like Problems

BOŠTJAN BREŠAR, MICHAEL A. HENNING, AND SANDI KLAVŽAR
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In this talk we consider $\{k\}$-dominating functions in graphs (or integer domination as we shall also say) that was first introduced by Domke, Hedetniemi, Laskar, and Fricke. For $k \geq 1$ an integer, a function $f\colon V(G) \to \{0, 1, \ldots, k\}$ defined on the vertices of a graph G is called a $\{k\}$-dominating function if the sum of its function values over any closed neighborhood is at least k. The weight of a $\{k\}$-dominating function is the sum of its function values over all vertices. The $\{k\}$-domination number of G is the minimum weight of a $\{k\}$-dominating function of G. We studied the $\{k\}$-domination number on the Cartesian product of graphs, mostly on problems related to the famous Vizing's conjecture. Three generalizations of the inequality by Clark and Suen will be presented.

Planar embeddability of the vertices of a graph using a fixed point set is NP-hard

SERGIO CABELLO
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Let G be a graph with n vertices and let P be a set of n points in the plane. We show that deciding whether there is a planar straight-line embedding of G such that its vertices are embedded onto the points P is NP-complete, even when G is 2-connected and 2-outerplanar. This settles an open problem posed by Bose, by Brandenberg et al., and by Kaufmann and Wiese.

A Semidefinite Programming Based Heuristic for Graph Coloring

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A *k-coloring* of a simple undirected graph $G(V, E)$ is a mapping $x : V \to \{1, ..., k\}$ such that

$$(ij) \in E \Rightarrow c(i) \neq c(j) \tag{1}$$

This traditional graph (vertex) coloring representation induces permutation redundancy as $c \circ \pi$, where π is any permutation of numbers $1...k$, is also a k-coloring *symmetric* to c. We see that the graph coloring problem is in fact a partitioning problem. Therefore we introduce a *coloring relation* R_c defined by

$$iR_cj \iff c(i) = c(j)$$

Any relation R is *represented* by a 0-1 matrix X defined by

$$x_{ij} = 1 \iff iRj \tag{2}$$

This matrix is positive semidefinite if and only if it represents an equivalence relation, and coloring relation is such. Denote by J the matrix of all ones, and let X represent any symmetric, homogeneous relation on the vertex set V satisfying (1). Then

$$lX \succeq J \iff l \geq k \,\&\, \text{matrix } X \text{ represents a } k\text{-coloring}$$

with a straightforward corollary

$$\chi(G) = \min\{l : lX \succeq J, X = X^T, x_{ii} = 1\, \forall i \in V, x_{ij} = 0\, \forall (ij) \in E, xij \in \{0, 1\}\}$$

By dropping the integrality condition $x_{ij} \in \{0, 1\}$ in this NP-hard problem we obtain Lovász theta number

$$\theta(\bar{G}) = \min\{l : lX \succeq J, X = X^T, x_{ii} = 1\, \forall i \in V, x_{ij} = 0\, \forall (ij) \in E\} \tag{3}$$

a well known polynomial lower bound on the chromatic number of a graph. Let X^* be a matrix at which an optimum of (3) is attained. Then (2) suggests that a large element $x_{ij} \doteq 1$ should be interpreted as color vertices i and j with the same color. Since $\theta(\bar{G})$ and X^* can be computed to any fixed precision in polynomial time by semidefinite programming, this idea motivates a Karger-Motwani-Sudan like recursive heuristic for graph coloring.

References

[1] P. GALLINIER and J.K. HAO. Hybrid evolutionary algorithms for graph coloring. *Journal of Combinatorial Optimization*, 3:379–397, 1999.

[2] D. KARGER, R. MOTWANI, and M. SUDAN. Approximate graph coloring by semidefinite programming. *Journal of the Association for Computing Machinery*, 45:246–265, 1998.

[3] L. LOVÁSZ. On the Shannon capacity of a graph. *IEEE Trans. Inform. Theory*, 25:1–7, 1979.

Θ-graceful labelings of partial cubes

Boštjan Brešar and Sandi Klavžar
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The Ringel-Kotzig conjecture asserting that all trees are graceful remains one of the central problems in the area of graph labelings. We introduce Θ-graceful labelings of partial cubes as a natural extension of graceful labelings of trees. Several classes of partial cubes are Θ-graceful, for instance even cycles, Fibonacci cubes, and (newly introduced) lexicographic subcubes. The Cartesian product of Θ-graceful partial cubes is again such and we wonder whether in fact any partial cube is Θ-graceful. A connection between Θ-graceful labelings and representations of integers in certain number systems is also established.

On edge-preserving maps of graphs

Wilfried Imrich
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A contraction of a graph G is a mapping $f : V(G) \to V(G)$ that preserves or contracts edges, that is, whenever $x, y \in E(G)$, then $f(x) = f(y$ or $f(x)f(y) \in E(G)$. Contractions are also known as weak endomorphisms or edge-preserving maps and have a plentyful variety of fixed subgraph properties. Let us just mention the well known fact that not only every automorphism, but also every contraction of a finite tree fixes a vertex or stabilizes an edge.

These results have been generalized to infinite graphs by Bandelt, Chastand, Halin, Polat, Quilliot, Sabidussi, and Tits mainly with the aim to find conditions that ensure the existence of a finite fixed subgraph rather than *fixed points at infinity.*

It is the aim of this talk to convey the spirit of the basic results and to add one about infinite median graphs.

Faber-Krahn Type Inequalities for (Non-regular) Trees

Josef Leydold
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The Faber-Krahn theorem states that among all bounded domains with the same volume in R^n (with the standard Euclidean metric), a ball that has lowest first Dirichlet eigenvalue. Recently it has been shown that a similar result holds for (semi-)regular trees. In this article we show that such a theorem also hold for other classes of (not necessarily non-regular) trees. However, for these new results no couterparts in the world of the Laplace-Beltrami-operator on manifolds are known.

Additive and hereditary properties of systems of objects

Peter Mihók

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We use the basic elementary notions of category theory. A concrete category $\mathbf{C}$ is a collection of *objects* and *arrows* called *morphisms*. An object in a concrete category $\mathbf{C}$ is *a set with structure*. We will denote the *ground-set* of the object A by $V(A)$. The morphism between two objects is a *structure preserving mapping*. Obviously, the morphisms of $\mathbf{C}$ have to satisfy the axioms of the category theory. The natural examples of concrete categories are: **Set** of sets, **FinSet** of finite sets, **Graph** of graphs, **Grp** of groups, **Poset** of partially ordered sets with structure preserving mappings, called homomorphisms of corresponding structures.

For example, a simple finite hypergraph $H = (V, E)$ can be considered as a system of its hyperedges $E = \{e_1, e_2, \ldots, e_m\}$, where edges are finite sets and the set of its vertices $V(H)$ is a superset of the union of hyperedges, i.e. $V \supseteq \bigcup_{i=1}^{m} e_i$. The following definition gives a natural generalization of graphs and hypergraphs.

Let $\mathbf{C}$ be a concrete category. A *simple system of objects* of $\mathbf{C}$ is an ordered pair $S = (V, E)$, where $E = \{A_1, A_2, \ldots, A_m\}$ is a finite set of the objects of $\mathbf{C}$, such that the ground-set $V(A_i)$ of each object $A_i \in E$ is a finite set with at least two elements (i.e. there are no loops) and $V \supseteq \bigcup_{i=1}^{m} V(A_i)$. The class of all simple systems of objects of $\mathbf{C}$ will be denoted by $\mathcal{I}(\mathbf{C})$. The symbols K_0 and K_1 denotes the null system $K_0 = (\emptyset, \emptyset)$ and system consisting of one isolated element, respectively. We will assume that by renaming (relabeling) the elements of the object A only, we obtain always an object A^* isomorphic to A in every concrete category $\mathbf{C}$l

For example, graphs can be viewed as systems of objects of a concrete category of two-element sets with bijections as arrows, digraphs are systems of objects of the category of two-element posets, hypergraps are finite set systems i.e. $\mathcal{I}(FinSet)$, etc. There are nice applications of systems of objects in information systems and computer science. A WAN network is a system on LANs, Internet is a system of WANs and the isomorphism in the category of LANs can be defined in a different way depending on the user requirements. The elements of the LANs are obviousely computers. Let us remark, that the L-structures generalizing graphs, digraphs and k-uniform hypergraphs are special systems of objects on category of relational structures.

To generalize the results on generalized colourings of graphs to arbitrary simple systems of objects we need to define *isomorphism of systems*. We can do this in a natural way: Let $S_1 = (V_1, E_1)$ and $S_2 = (V_2, E_2)$ be two simple systems of objects of a given concrete category $\mathbf{C}$. The systems S_1 and S_2 are said to be isomorphic if there is a pair of bijection:

$$\phi : V_1 \longleftrightarrow V_2; \qquad\qquad \psi : E_1 \longleftrightarrow E_2,$$

such that if $\psi(A_{1i}) = A_{2j}$ then $\phi/V(A_{1i}) : V(A_{1i}) \longleftrightarrow V(A_{2j})$ is an isomorphism of the objects $A_{1i} \in E_1$ and $A_{2j} \in E_2$ in the category $\mathbf{C}$. The homomorphism of the systems can be defined in a similar way.

The disjoint union of the systems S_1 and S_2 is the system $S_1 \cup S_2 = (V_1 \cup V_2, E_1 \cup E_2)$, where we assume that $V_1 \cap V_2 = \emptyset$. A system is said to be connected if it cannot be expressed as a disjoint union of two systems.

The subsystem of S_1 induced by the set $U \subseteq V(S_1)$ is $S_1[U]$, with objects $E(S_1[U]) := \{A_{1i} \in E(S_1) | V(A_{1i}) \subseteq U\}$. S_2 is an induced-subsystem of S_1 if it is isomorphic to $S_1[U]$ for some $U \subseteq V(S_1)$.

Using these definitions we can define, analogously as for graphs, that an additive induced-hereditary property of simple systems of objects of a category $\mathbf{C}$ is any class of systems closed under isomorphism, induced-subsystems and disjoint union of systems. Let us denote by $\boldsymbol{MM}^a(\mathbf{C})$ the set of all additive induced-hereditary properties of simple systems of objects of a category $\mathbf{C}$.

In our talk we will consider the structure of additive hereditary properties of systems of objects.

On Almost-Median Graphs

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A short history of almost-median graphs will be represented. Following with two nice families of planar almost-median graphs and a new characterization of almost-median graphs.

How to approximate the bandwidth of a graph using semidefinite programming?

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The bandwidth problem is an old problem from combinatorial optimization, where, given a graph $G = (V,\ E)$, one looks for such a labelling $\Phi : V \to \{1,\ 2,\ ...,|V|\}$, which minimizes a maximal distance $\sigma_\infty(G, \Phi) = \max_{(uv)\in E} |\Phi(u) - \Phi(v)|$ over all possible labellings. Let denote $\sigma_\infty(G) = \min_\Phi \sigma_\infty(G, \Phi)$. The problem how to find the $\sigma_\infty(G)$ or the optimal labelling is proven to be NP-hard problem. In the year 2000 Blum et al. proposed in [1] the semidefinite relaxation of this problem and suggested the rounding scheme, which gives the labeling Φ^* with $\sigma_\infty(G, \Phi^*) \leq O(\sqrt{n/b}\log n)\sigma_\infty(G)$, where b is the optimal value of the semidefinite relaxation.

They argued its polynomial time complexity with ellipsoidal method, so the result seemed to have only a theoretical value. We're going to present heuristics for solving this SDP relaxation, which relies on the interior point methods and on the bundle method and gives good results in practice (much better than the theoretical guaranty does), although I can prove its polynomial time complexity only for some small classes of graphs. We're also going to present how to transform the proposed randomized algorithm into deterministic one on some specific classes of graphs (e. g. caterpillars) and how good results do we get this way.

Bibliography:

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Transitive Digraphs

Norbert Seifter
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For decades undirected transitive graphs have been a topic of deep investigations. Although many of the methods developed for undirected graphs can also be applied to undirected graphs (digraphs), the problems arising in the context of digraphs are quite different. In this talk we present some of these problems.

Characterisation of the Resonance Graphs of Catacondensed Hexagonal Graphs

Aleksander Vesel
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The vertex set of the resonance graph of a hexagonal graph G consists of 1-factors of G, two 1-factors being adjacent whenever their symmetric difference forms the edge set of a hexagon of G. A characterization of the resonance graphs of catacondensed hexagonal graph is presented. The characterisation is the basis for the algorithm that recognizes the resonance graph of a catacondensed hexagonal graph. Moreover, the modified algorithm can be applied for recognizing the Fibonnacci cubes.

Weak Reconstruction of Strong Product Graphs

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We prove that the class of nontrivial connected strong product is weakly reconstructible. We also show that any nontrivial connected thin strong product graph can be uniquely reconstructed from each of its one-vertex-deleted deleted subgraphs.

Fibonacci cubes are the resonance graphs of fibonaccenes

Petra Žigert and Sandi Klavžar
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Fibonacci cubes were introduced in 1993 and intensively studied afterwards. Fibonacci cubes are precisely the resonance graphs of fibonaccenes. Fibonaccenes are graphs that appear in chemical graph theory and resonance graphs reflect the structure of their perfect matchings. Some consequences of the main result will also be presented.

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